DOCUMENT RESUME

ED 050 963 SE 011 252

TITLE The Improvement of Mathematics Instruction in Oklahoma, Grades K-12.

INSTITUTION Oklahoma Curriculum Improvement Commission, Oklahoma City.; Oklahoma State Dept. of Education, Oklahoma City. Div. of Special Education.

PUB DATE 69

NOTE 103p.

EDRS PRICE EDRS Price MF-$0.65 HC-$6.58

DESCRIPTORS *Curriculum, *Curriculum Guides, Elementary School Mathematics, *Instruction, Modern Mathematics, Secondary School Mathematics, *Teacher Education

IDENTIFIERS Oklahoma

ABSTRACT

The following topics are discussed in this state handbook: (1) characteristics of a modern math program (structure, vocabulary, correctness); (2) teacher preparation (program descriptions and source material lists for inservice education and the CUPM recommendations for training math teachers); (3) initiating a modern program in mathematics in a particular school system (programs for different types of students, resources for administrators, guidelines for improving programs); (4) mathematics for the underachiever (content suggestions, objectives, teachers of low achievers, experimental programs, high school general mathematics); and (5) mathematics for the talented student (resources, club activities, content topics, suggestions for the disinterested student). (JG)

ED050963

The Improvement of Mathematics Instruction In Oklahoma Grades K-12

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

Tentative Recommendation of
The State Mathematics Committee
of
THE OKLAHOMA CURRICULUM
IMPROVEMENT COMMISSION

1907

SE 011 252

THE OKLAHOMA STATE DEPARTMENT OF EDUCATION
D. D. Creech, Superintendent
1969

ED050963

The Improvement of Mathematics Instruction In Oklahoma Grades K-12

Tentative Recommendation of the State
Mathematics Committee
of
THE OKLAHOMA CURRICULUM IMPROVEMENT COMMISSION
W. D. Carr, Chairman
Clifford Wright, Executive Secretary

THE OKLAHOMA STATE DEPARTMENT OF EDUCATION

D. D. Creech, Superintendent

1969

FOREWORD

The mathematics program has been undergoing rapid changes during the last few decades, yet the major theories of two thousand years ago are still valid. Technological advances in our society require every person to have a better background in mathematics. The mathematics program should provide each student, according to his ability, with an understanding of mathematics adequate for his current and future needs.

The program of mathematics in the schools must be modernized in such a way that our children can understand the basic principles involved and become acquainted with new concepts of great importance in the modern world. Instruction should involve periods of exploration and experimentation leading toward generalization. The concepts should be introduced early. Patterns of course offerings will vary in individual schools. The failure to make mathematics relevant to modern day living is probably one of the reasons that it is not grasped with the same degree of enthusiasm by students as are other courses. The student should become aware of the power and influences of mathematics in his continuing vocational and personal development.

The State Department of Education gratefully acknowledges the contribution of the members of the State Mathematics Committee of the Oklahoma Curriculum Improvement Commission who prepared this guide. It is my hope that this will prove to be a useful resource for all teachers in the area of mathematics.

D. D. Creech
State Department of Public Instruction

ACKNOWLEDGEMENTS

The State Committee on Mathematics has been constantly at work on the continuous improvement of the teaching of mathematics. I wish to express our sincere appreciation for the able efforts of this Committee, especially to the Chairman, Miss Eunice Lewis and to Dr. James H. Zant, who has served as editor of this guide. This material should be a useful resource for all teachers of mathematics.

Clifford Wright, *Director of Curriculum*
State Department of Education

Membership of the State Committee

Dr. Milton Berg, *professor*
Oklahoma State University

Mrs. Oma M. Borchardt, *teacher*
Jackson Elementary, Norman

Dr. Sarah Burkhart, *supervisor*
Tulsa Public Schools

Mr. W. J. Coffia, *research associate*
Oklahoma City Public Schools

Mr. Don Connel, *mathematics supervisor*
State Department of Education

Membership of the State Committee (continued)

Mr. Roy Daniels, *professor*
Phillips University, Enid

Mrs. Sarah Duke, *teacher*
Sulphur Public Schools

Mrs. Dale Edmond, *supervisor*
Tulsa Elementary Schools

Mr. Robert Harvey, *professor*
Northeastern State College, Tahlequah

Dr. Harold Huneke, *professor*
The University of Oklahoma

Mrs. Bettye Jewell, *teacher*
Holland Hall School, Tulsa

Miss Eunice Lewis, *CHAIRMAN, professor*
The University of Oklahoma, Norman

Dr. Jack Phelps, *professor*
Southwestern State College, Weatherford

*Mr. Coy C. Pruitt, *supervisor*
Public School Mathematics, Tulsa

*Mrs. Bessie Sanmann, *teacher*
Lawton Public Schools

*Miss Marie Schurch, *teacher*
Henryetta Public Schools

Miss Edith Steanson, *teacher*
The University of Oklahoma Laboratory School

*Mrs. Nina Thayer, *teacher*
Skiatook Public Schools

Mr. Leslie Waid, *coordinator of mathematics*
Muskogee Public Schools

Dr. James H. Zant, *editor, professor emeritus*
Oklahoma State University

Membership of the State Committee (continued)

Mr. Kenneth Culver, *assistant executive secretary*
Oklahoma Curriculum Improvement Commission

Mrs. Ernestine Fisher, *assistant executive secretary*
Oklahoma Curriculum Improvement Commission

Dr. Clifford Wright, *executive secretary*
Oklahoma Curriculum Improvement Commission

***Resigned**

(This publication is made possible by funds from Title V, Section 503, of the Elementary and Secondary Education Act of 1965 through the Curriculum Division of the Oklahoma State Department of Education.)

CONTENTS

FOREWORD iii

ACKNOWLEDGMENTS v

MEMBERSHIP OF THE STATE COMMITTEE v

INTRODUCTION 1

SECTION I

Characteristics of a Modern Program in Mathematics for Oklahoma Schools 5

SECTION II

Ways of Preparing Teachers in a Particular School System to Use New Mathematics 19

SECTION III

Initiating a Modern Program in Mathematics in a Particular School System 39

SECTION IV

Mathematics for the Underachiever 51

SECTION V

Mathematics for the Talented Student 77

SECTION VI

General Bibliography 93

THE IMPROVEMENT OF MATHEMATICS INSTRUCTION IN OKLAHOMA SCHOOLS K-12

Introduction

The last handbook, *The Improvement of Mathematics Instruction in Oklahoma K-12,* was published by the Oklahoma State Department of Education in December, 1960 and has been out of print for several years. Since it was developed at the beginning of a period of transition in this state from a traditional program in mathematics to one which is called modern mathematics and which involves an emphasis on understanding, meaning and structure, much of the material included is not pertinent to the present day situation in the schools of the state.

During the period 1959-1964 Oklahoma made a considerable use of experimental programs and textbooks in the schools and played an important role in the development of the new mathematics for the schools. Most of the teaching material used was that developed by the School Mathematics Study Group (SMSG), though other new material was also used. The use of this material was widespread over the state, but at no time did it involve more than twenty to thirty per cent of the school children, grades one through twelve. The various educational agencies over the state were able, during this period, with the aid of NSF Institutes, summer programs in nearly all of the colleges and inservice programs in many schools, to prepare the teachers involved in these experimental programs and some others to the extent that they were reasonably competent to teach this material in the schools. However, there were many teachers in the state who were not involved in these experimental programs and who were not able to prepare themselves for teaching mathematics with the new emphasis on understanding, concepts and structure.

As was to be expected, the textbooks adopted for the period 1964-1968 contained many of the new ideas in mathematics, though in varying degrees.

Teachers who had been involved in the experimental programs and a few others who had taken advantage of the various institutes and inservice programs had little trouble in adapting themselves to the use of the newly adopted textbooks. But those who had not been so involved were in trouble and there was too little help available to them. This problem has been and is particularly present with elementary school teachers. Traditionally such teachers were not required to study any mathematics in college and little in high school.

Apparently, these inadequately prepared teachers are those employed in the small dependent school districts over the state and certainly many of them are in such districts. However, it may also be true that even in larger districts, probably less than thirty per cent of the children were involved in the experimental programs, so that many teachers in these districts were also inadequately prepared to use the new textbooks.

It is anticipated that new textbooks adopted in the future will include many more of the new concepts and ideas than those of the present adoption. There is evidence of this from the new editions which are now on the market. Because of this situation in the state, this handbook will have as its central feature suggestions which will help teachers and administrators at the individual school level to meet the problems of teaching new mathematics which already exist and others which will surely be identified when new textbooks are adopted.

The following topics are discussed in this handbook:

I. *Characteristics of a modern program in mathematics for Oklahoma schools.*

A new program in mathematics or as it is sometimes called, Modern Mathematics, is often misunderstood. The Committee in this section has explained the meaning of the modern program as clearly as possible. The content and organization of the program is presented by means of questions, discussions and illustrative examples dealing with specific characteristics of the content as contrasted to chapter headings and topics of instruction.

II. *Ways of preparing the teachers in a particular school system to use the new mathematics.*

This is the most difficult problem faced by the school. While not all teachers need this preparation, many of them do; all are faced with the necessity of keeping abreast in their field. The Committee feels that a knowledge and fundamental understanding of the basic principles of mathematics must be the first step in this procedure. It is something for which administrators and teachers must prepare.

III. *Initiating a modern or new program in mathematics in a particular school.*

This is basically an administrative problem, but teachers must also be involved. The two topics listed above are of course, related to initiating a new program. However, the selection of a mathematics program for a school must also involve the type of community it serves and the needs of the individual students in the school.

IV. *Mathematics for the underachiever.*

What has been presented here is largely conjecture, since little is known about mathematics for this group or how it should be taught and what its values are. Material is being developed for this type of student and teachers must carefully evaluate these efforts.

V. *Mathematics for the talented student.*

Talented students in the schools do not usually work up to capacity. They need materials which cannot be included in textbooks, but which is readily available in many other places. Teachers must also be dedicated, interested and competent in many areas of mathematics which are not involved when teaching average or slow learning students. Many suggestions are included here, but it is likely that the students will be interested in a number of other topics which are not listed.

VI. *General bibliography.*

As the list of topics discussed in this handbook indicate, it can be seen that the Committee hopes to help and influence the teachers and administrators of Oklahoma in dealing with pertinent problems which arise from a continued period of transition from a traditional program to a modern program in mathematics in the schools. This transition is incomplete now, both from the point of view of the content in mathematics which should be presented, its organization or sequence in the school program and the methods of presenting it to the students. Therefore, it is not felt that the mathematical and educational community is in any position to present to the schools a scope and sequence recommendation for grades one through twelve. There are too many unanswered questions at this time and new experimentation is going on all of the time.

This handbook has been developed by several small writing committees made up from members of the State Committee on Mathematics and others.

The entire committee has been kept informed about the progress and activities of the writing committees and they have made valuable criticism and suggestions.

References to books, magazines and films are made throughout the discussion of the various topics. This is done to indicate to the reader where other discussions of the particular topic may be found. These references are made by author and title only. However, a complete listing of the material referred to, and other material as well, is included in a bibliography at the end of the section. This listing includes author, title, publisher, address and date of publication.

SECTION I

Characteristics of a Modern Program in Mathematics for Oklahoma Schools

Introduction

It is true that the mathematics being taught in the schools is a rapidly changing subject and that the phrase "New Mathematics" is coming into popular use. Teachers must know what changes have been made and how they will affect the teaching of mathematics at all levels. Although a number of topics such as: set, equations, inequalities and probability, have been added to present day textbooks and school programs in mathematics, most of the topics which were previously included are still there. In the elementary school these are such topics as: numeration, addition, subtraction, multiplication, division, decimals, measurement and problem solving. In the secondary school such topics as: factoring, equations, multiplication with algebraic expressions, circles in geometry, square root and radicals are included in the new textbooks as well as in the old ones.

In designing a modern program in mathematics teachers must realize that most of the changes which have occurred involve a point of view toward teaching and learning mathematics rather than the study of new topics and subject matter. It is recommended now that teachers teach and students learn mathematics from the point of view of understanding, a broader vocabulary, correctness, more concise definition, and some idea of the structure of mathematics and the basic concepts of number and geometry. New concepts, when they are used, are for the purpose of helping children to learn mathematics from this new point of view. For example, the ideas of sets and operations on them are introduced because they help students to understand such things as the basic meanings and principles of operations on numbers. Inequalities and graphs are studied because these topics give a more complete concept of the equation and its uses in solving problems. It is qualities such as these which teachers and administrators *must* use in making value judgments about what to include in the mathematics program for the schools.

The Content and Organization of the Program in Mathematics for the Schools

I. *Preliminary Considerations*

Since schools in Oklahoma vary widely in the type of community served, the contribution of mathematics to the total educational program of the schools and to individual students, competence of teachers and students and the like, each school must consider and arrive at a consensus as to:

1. The contribution of mathematics to the total educational program of the school and to the individual students;
2. The philosophy of the school system and the direction to be taken by its program;
3. The mathematical competence of the teachers who will be using the text (Section II of this guide will deal with the problem of

improving the competence of the staff of a school in the area of mathematics);

4. The variety of mathematics programs available and the range of the student's abilities. (For example, other sections of this guide will deal with possible mathematics programs for the low achievers and for talented students).

The formulation of a mathematics program for a particular school system is the joint responsibility of the teachers and administrators of the system. The program should be formalized prior to the investigation of possible textbooks and other teaching materials. The program for a particular school and the means of presenting it to the students is more likely to satisfy the needs of the school if opinions of the teachers who are to use the materials are considered. It is felt that all of these steps should be taken prior to an active investigation of texts and that this procedure will result in a better program.

II. *Criteria Related to the Presentation and Content of Mathematics in the Elementary School, Grades K-6*

This will be discussed under the following headings:

1. Structure, 2. Vocabulary, 3. Definitions, 4. Correctness, 5. Generalizations, 6. Sequence of Topics, 7. Tests, Exercises and Reviews, 8. Illustrative Examples and 9. Teachability. The discussion of these topics will include short discussions of the meanings of the topic when necessary. Any questions concerning the presentation and content of the topic, and illustrations in the form of examples designed to clarify the presentation and/or content will also be discussed.

1. *Structure*

Mathematics at the elementary school level, and at other levels as well, is made up of a connected body of knowledge. A particular part depends upon what has gone before and on the fundamental properties of the subject. The whole system is tied together with relationships and a logical development. This organized body of knowledge, which actually involves words, definitions, assumptions or basic principles, such as the commutative property, and proofs based on the assumptions and definitions, is often called the structure of mathematics.

1) Does the presentation assist the student in understanding the structure of this particular area of mathematics?

Example: Early in school, children are able to discover that with the set of whole numbers 0, 1, 2, 3, . . . they are able to add and to multiply any of these and get a result which is a member of the set, that is, a whole number. We teach them to say that the set of whole numbers is closed under the operation of addition and multiplication. This is a basic principle or property of the set of whole numbers and becomes a part of the *structure* of the organized body of knowledge, which is mathematics. However, the pupil may also discover at the same time that $8 \div 3$ is not a whole number and may then make the statement that "The set of whole numbers is *not closed* under the operation of division."

Example: If the commutative property has been verified for addition, then is the student asked to test this property for other operations?

2) As an extension of a topic is made, does the development show clearly how the extension is related to the structure under consideration?

   Example: Using the identity element for multiplication to rename rational numbers or fractions.

3) Is the development of the topic made on appropriate levels of understanding?

   Example: To what extent are reasons presented for principles and procedures? Are children asked to show that the set of rational numbers is closed under the operation of division?

2. *Vocabulary*
   1) Is the vocabulary appropriate for the level of the student?
   2) Is the rate of introducing new terms appropriate to the mathematical maturity of the student?
   3) Once a term has been defined, is it used throughout the book or series?

3. *Definitions*
   1) Do all definitions contain in the defining expression only those terms which can reasonably be expected to be understood by the student at his state of development in the subject?

      Example: If "simple closed curve" is used to define a polygon, is "simple closed curve" understood?

   2) Do definitions include the parts necessary to understand the term being defined and also enough to distinguish the object from similar ones?

      Example: "A square is a closed curve consisting of four congruent segments" is a true statement, but it is not enough to distinguish between the terms "square" and "rhombus" and hence does not define a square. One must add "and the line segments form right angles."

      Example: "A triangle is a simple closed curve formed by three line segments" is not a satisfactory definition. One must add, "the sum of the lengths of any two sides is greater than the length of the third side."

   3) Has the definition been so written that the defining expression does not contain the term being defined, or a term directly derived from it, which has not been defined?

Example: "A circle is a round ring" uses the word "ring", a synonym for "circle." (Correct definition is: A circle is a closed curve all of whose points are the same distance from a point in the interior called the center.)

4) Does the content make clear the particular usage of a term which may have a different meaning in another context?

Example: "Divide evenly" as it relates to sharing as opposed to a division problem where the remainder is zero.

4. *Correctness*

1) Is the material free of statements most mathematicians would agree are false?

Example: Implying there are more points on a line segment 10 units long than on a line segment one unit long. Confusing the disc enclosed by a circle with the circle. Using the expression "the area of a circle or a polygon" instead of "the area of the *interior* of a circle, and the area of the *interior* of a polygon."

5. *Generalizations*

Generalizations in elementary school mathematics are usually the result of a few specific examples and the feeling on the part of the pupil that he has discovered a pattern which will always recur. After a few trials, such as, $2 + 4 = 6$, $8 + 4 = 12$ and $24 + 26 = 50$, he may generalize that the sum of two even numbers is an even number. This is not mathematically sound, since it is an inductive argument, but in the hands of a skillful teacher pupils may be led to do some careful thinking and also recognize that such procedures have limitations. Proofs are probably too difficult at this level, but careful thinking will help them in proofs later on.

One must be careful that students do not make generalizations which will hinder them at a later time. A common mistake is for people to give the answer 4 to the problem $16 \div \frac{1}{4} = \_\_$, because they have made the generalization, perhaps unconsciously, that when you divide the result is always less than the dividend.

1) Are opportunities to generalize provided?
Example: If $5 \times 9 = 45$, then is it true that any number whose decimal numeral ends in 0 or 5 is divisible by 5?

2) Are instances that lead to generalizations appropriate in sequence and number?

3) Does the author attempt to cultivate and capitalize on the reader's intuitive understanding while pointing out the dangers and limitations of dependence on intuition?

Example: Are children led to see that multiplication does not always result in a larger number than the multiplicand (i.e., $64 \times \frac{3}{4} = 48$; $75 \times .53 = 39.75$).

Example: Are children led to see that division does not always result in a number smaller than the dividend? (For example, $25 \div 1/5 = 125$, $27 \div .75 = 36$).

4) Is it made clear that the demonstration of many instances of a mathematical generalization does not prove it, although it does increase its plausibility?

   Example: Testing the divisibility of a number by three by dividing the sum of its digits by three for many numbers does not prove the rule. (Reread 5, 1)

   (Elementary school children can prove this rule by writing with understanding the following sequence:)

$$\begin{aligned} 273 &= 2(100) + 7(10) + 3 \\ &= 2(99 + 1) + 7(9 + 1) + 3 \\ &= 2(99) + 2(1) + 7(9) + 7(1) + 3 \\ &= [2(99) + 7(9)] + [2 + 7 + 3] \\ &= 2 \times 33 \times 3 + 7 \times 3 \times 3 + 12 \\ &= [66 \times 3 + 21 \times 3] + 12 \\ &= 3(66 + 21) + 12 \end{aligned}$$

5) Are counter-examples used whenever they may be employed to advantage?

   Example: $7 - 9 = -2$ and $16/3 = 5\frac{1}{3}$ show by counter-example that the set of whole numbers is not closed under the operation of subtraction and division. (For example, -2 and $5\frac{1}{3}$ are not included in the set of whole numbers.)

   Example: $8 - 7$ is not equal to $7 - 8$ is a counter-example which shows that the operation of subtraction is not commutative.

6. *Ordering and Sequence of Topics*

   1) Is the material presented in a way which develops various topics on an elementary level and returns to the development at intervals exploring each topic more deeply with each repetition?

      Example: The commutative property, encountered first with whole numbers, also holds for rational numbers, i.e., $\frac{2}{3} + \frac{3}{4} = \frac{3}{4} + \frac{2}{3}$, for decimals, i.e., $3.6 + 7.2 = 7.2 + 3.6$, for negative numbers, i.e., $2 + (-7) = (-7) + 2$, etc.

   2) Are basic concepts developed in simple cases before they are used to explain more complicated ones?

      Example: Is the idea of the measure of a line segment well understood before the idea of the measure of the perimeters of triangles and polygons or the circumference of a circle are introduced?

7. *Tests, Exercises, and Reviews*

   1) Are there adequate materials to permit a valid evaluation?

   2) Are there materials to help the student develop the power and habit of self-evaluation?

3) Are there periodic tests to assist in diagnosis and evaluation?

4) Are questions clearly and concisely stated to avoid being misinterpreted?

5) Are there some exercises that require the student to generalize, to discover, to consolidate concepts, to improve skills and to apply what he has learned to new situations?

6) Are exercises of varying difficulty identified?

7) Are there exercises designed to challenge the student?

8) Do the review exercises make it possible to identify specific instructional needs?

9) Following the "identification of specific instructional needs" does the text provide for the correction of these weaknesses?

10) Does the review direct attention to the significant elements and their relationship to each other?

11) Is there sufficient practice in mixed problem types so that the pupil has opportunity to identify the process without reference to the page title?

8. *Illustrative Examples*

1) Do the examples clarify the concepts presented?

2) Are examples used to lead into similar problems in a set of exercises without being merely duplications?

3) Are examples used to indicate the area of application of generalization?

4) Are examples appropriate for the grade level?

9. *Teachability*

1) Are ideas developed by raising questions, considering alternatives and encouraging conjectures which will be verified later?

2) Are needed readiness references included before the introduction of new concepts so that the student may see the problem in a better perspective?

3) Is the material written and organized in such a manner that those items considered to be optional can be deleted without destroying the continuity of the presentation?

4) Are the limitations and dangers of intuitive understandings and generalizations pointed out? Many specific instances do not necessarily imply a true statement.

III. *Criteria Related to the Presentation and Content of Mathematics in the Secondary School, Grades 7-12*

The secondary school criteria will be discussed under headings, many of which are similar to those used in the discussion of Criteria for the Elementary School. However, the instructional material will be more diversified and will be presented from a higher level. The headings are: 1. Structure, 2. Rigor, 3. Vocabulary, 4. Definitions, 5. Correctness, 6. Theorems and Proofs, 7. Sequence of Topics, 8. Generalizations, 9. Tests, Exercises and Reviews, 10. Illustrative Examples, 11. Teachability, and 12. Optional Topics. Discussions, questions and examples will be used as they were in II.

1. *Structure*

1) Does the presentation assist the student in understanding the structure of this particular area of mathematics?

   Example: Is it pointed out that $y = 3x + 6$ describes a set of ordered pairs, determines a function and defines a line?

2) As an extension of a topic is made, does the development show clearly how the extension is related to the structure under consideration?

   Example: As the whole numbers are extended to real numbers, does the material show that the multiplicative identity exists in each extension?

2. *Rigor*

Rigor in a course of study refers to the nature of the development of the arguments and the kind of justification that is used in proof. Few presentations are entirely rigorous or completely without rigor. The level of rigor useful in teaching varies with the level of the class and the maturity of the students. The Pythagorean Theorem is discussed intuitively in Grade 7, but a more rigorous proof is given later. The level of rigor in a text may have much to do with the future understanding of the subject by its reader.

1) Is the development of each topic made on an appropriate level of rigor?

   Example: To what extent are reasons included for principles and procedures?

2) Is an attempt made to cultivate and capitalize on the reader's intuitive understanding while pointing out the dangers and limitations of dependence on intuition?

   Example: Since $1 + 3 = 4$, $1 + 3 + 5 = 9$, $1 + 3 + 5 + 7 = 16$; does it appear that $1 + 3 + 5 + 7 \ldots + (2n - 1) = n^2$? Are proofs offered for some of these conjectures?

3) Is the material presented in such a way that the student is expected to make conjectures, test their truth in specific instances and prove some of them?

   Example: The student learns or proves that the diagonals of a rectangle bisect each other. Does this condition hold for diagonals of all quadrilaterals? If not, does it hold for any quadrilaterals other than rectangles?

4) Are the proofs appropriate for the maturity and background of the students?

Example: At what level should a proof by mathematical induction be used? What is the appropriate level for the use of the indirect method of proof?

5) If insufficient reasons are given for some of the steps of a development, is this justified in the material for the student and are the omissions indicated?

Example: The Pythagorean Theorem should probably be accepted without a formal proof in Grade 7, but with an intuitive approach involving measurement or cutting the squares. However, in a later grade the proof would be required.

6) Does the material point out the danger of such common errors as circular proof, assuming truth of converses, assuming uniqueness and using theorems as evidence before they have been demonstrated?

Example: Trying to prove the base angles of an isosceles triangle congruent by drawing a line from the vertex to the mid-point of the opposite side without having proved that any line segment can be bisected. (To prove the latter one would need the theorem being attempted. Yet this proof appears in many textbooks. How do you prove it?)

3. *Vocabulary*

1) Is the vocabulary appropriate for the level and maturity of the student?

2) Is the rate of introducing new words and new terms appropriate to the mathematical maturity of the students?

3) Once a term or name has been defined, is it used?

4) In order to strengthen understanding, are ideas restated differently, perhaps in "the student's own words," rather than by verbatim repetition?

4. *Definitions and Undefined Terms*

1) Do the definitions in the course of study contain in the defining expression only those terms which can reasonably be expected to be understood by the student at his stage of development in the subject?

Example: Defining a quadrilateral as a geometric configuration consisting of four sides when geometric configuration is not understood.

2) For the object to be named by the term being defined, are the stated conditions necessary and sufficient?

Example: Defining a regular polygon as a polygon with congruent line segments for sides is not satisfactory. (It is necessary that the regular polygon have congruent line segments for sides, but this is not enough. The sufficient condition is that the polygon also have equal angles.)

3) Has the defining expression avoided using the term being defined or a term directly derived from it which has not been defined?

   Example: The definition, "A circle is a ring," is not a definition since the word "ring" is a synonym for circle and hence is a term derived from circle and not itself defined.

4) Does the content make clear the particular usage of a term which may have a different meaning in another context?

   Example: "Equivalent" as used with sets and compared to its use with equations.

5) Is there a clear distinction between terms taken as undefined and those defined?

   Example: The so-called definitions, "A point is that which has location only" and "A line segment is the shortest distance between two points" because they contain words like "location," "distance," "between," which cannot themselves be defined. One must start somewhere with words and "point," "line," "between," etc., are considered *undefined* terms.

5. *Correctness*

   Is the teaching material free of statements most mathematicians would agree are false?

   Example: Speaking of the area of a polygon or of a circle instead of the area of the interior.

   Example: Saying that $x^2 - 2$, for example, is not factorable without giving the domain of definition of the variable. If the domain is the set of real numbers, then the factors are $x - \sqrt{2}$ and $x + \sqrt{2}$.

6. *Theorems and Proofs*

   1) Is it made clear that the demonstration of many instances of a mathematical generalization does not prove it, although it does increase its plausibility?

      Example: "3, 5, 7, 11, 13, . . . are prime numbers; therefore all prime numbers are odd numbers," is not a valid statement since any number of cases do not prove a mathematical generalization. How about 2?

   2) Are the steps in the proofs sufficiently short so that the normal reader can bridge the gap?

   3) Are counter-examples used whenever they may be employed to advantage?

      Example: In $f(n) = n^2 - n + 41$, $f(0) = 41$, $f(1) = 41$, $f(2) = 43$, $f(3) = 47, \ldots f(40) = 1601$, all of them prime numbers. But try 41, $f(41) = 41^2 - 41 + 41^2 = 41 \times 41$, not a prime number. There are others, such as $f(82)$ $f(123)$, etc. Hence, the generalization that this function will always generate a prime number is not true, though it does so for the first 40 numbers tried.

4) Are all postulates which are used stated explicitly, including those which are plausible and often implicitly assumed?

5) Is it pointed out that there may be other proofs of a theorem?

   Example: The Pythagorean Theorem has more than 200 known proofs.

6) Is it quite clearly indicated in each case where a proof or demonstration ends?

7) Is appropriate logic available before proofs of theorems are introduced?

8) Has a foundation been laid for indirect proofs before this kind of proof is used in the text? Are fewer theorems presented for acceptance on faith, that is, as assumptions or postulates, as the course continues?

9) Is it made clear why the hypotheses of theorems are as stated and how the conclusion would be changed if one or more of the hypotheses were altered?

7. *Generalizations*

1) Are opportunities to generalize provided?

   Example: When certain geometric properties are established for two-space, are students encouraged to consider whether these properties can be generalized to three-space? "In a plane (2-space) one and only one line can be drawn perpendicular to a given line at a given point. In 3-space one and only one plane is perpendicular to a given line at a given point."

2) Are instances that lead to generalizations appropriate in sequence and number?

3) Are intermediate generalizations brought in when necessary to establish continuity in generalization?

8. *Sequence of Topics*

1) Is the material presented in a way which develops various topics spirally, that is, first on an elementary level and then returning to the development at intervals exploring each topic more deeply with each repetition?

   Example: Factorability of algebraic expressions depends on the universe or domain of the variable and $x^4 - 25$ factored over the integers is $(x^2 - 5)\ (x^2 + 5)$; over the reals is $(x - \sqrt{5})\ (x + \sqrt{5})\ (x^2 + 5)$ and finally over the complex numbers is $(x - \sqrt{5})\ (x + \sqrt{5})\ (x + i\sqrt{5})\ (x - i\sqrt{5})$.

2) Are reasons given for choosing any unusual arrangement or sequence of material or for a particular approach?

Example: When the indirect method of proof is used for the first time, are students informed about it and prepared for its use?

9. *Texts, Exercises, and Reviews*
    1) Are there adequate materials to permit a valid evaluation?
    2) Are there materials to help the pupil develop the power and habit of self-evaluation?
    3) Are there periodic tests to assist in evaluating?
    4) Do tests provide a means for the evaluation of the entire range of abilities?
    5) Are questions clearly and concisely stated to avoid their being misinterpreted?
    6) Are there exercises that require the student to generalize, to discover, to consolidate concepts, to improve skills, and to apply what he has learned to new situations?
    7) Are exercises of varying difficulty identified?
    8) Are there exercises designed to challenge the students?
    9) Are there exercises designed to encourage thinking?
    10) Do review exercises make it possible to identify specific instructional needs?
    11) Does the review direct attention to the significant elements and their relationship to each other?

10. *Illustrative Examples*
    1) Do the examples clarify the concepts presented?
    2) Are examples used to lead into similar problems in the set of exercises without being merely duplications?
    3) Are examples used to indicate the area of application or generalization?
    4) Are the implications of a generalization sometimes lost because examples of its application are not cited to indicate its extensiveness?
    5) Are examples appropriate for the grade level?

11. *Teachability*
    1) Are the ideas developed by raising questions, considering alternatives, and encouraging conjectures which may be verified later?
    2) Are the references made to topics which precede and follow so that the students may see the problem in better perspective?

3) Is the material written in such a manner that those items considered to be optional can be deleted without destroying the continuity of the presentation?

* * * * * *

It should be clear that the discussions and criteria included in this section do not form an attempt to establish a "state" curriculum in mathematics or imply the need for a scope and sequence study in this field. At all times the local community's characteristics and needs must be considered. The purpose here is to help a particular school in its decision-making process, not to make decisions for the school. Moreover, the mathematics curriculum for the schools is not yet stabilized to the point where this committee feels that the mathematics program at the state or national level can be properly written down. The matter of how children learn mathematics, the use and usefulness of newer media in the classroom and the whole field of modern educational technology are still uncertain elements in the educational field.

Bibliography

I. *Books:*

1. American Association of School Administrators, Association of Supervision and Curriculum Development, National Association of Secondary School Principals and the National Council of Teachers of Mathematics, *Administrative Responsibilities for Improving Mathematics Programs:* NCTM*, 1965, Washington, D.C.

2. Association of Supervision and Curriculum Development, *Commission on Current Curriculum Developments:* The Commission, 1965, pp. 45-56, Washington, D.C.

3. Deans, Edwina, *Elementary School Mathematics: New Directions,* U. S. Department of Health, Education and Welfare, Office of Education, Washington, D.C., G.P.O., 1963.

4. Educational Services, Inc., *Goals for School Mathematics: Report of the Cambridge Conference on School Mathematics,* New York: Houghton Mifflin Co., 1963.

5. Miles, Matthew B., Editor, *Innovations in Education:* Bureau of Publications, Teachers College, Columbia University, New York: 1964. See Chapter 14, p. 329, "School Superintendents and the Adoption of Modern Math: A Social Structure Profile."

6. National Council of Teacher of Mathematics, *An Analysis of New Mathematics Programs:* The Council, Washington, D.C., 1963.

7. NCTM, *The Growth of Mathematical Ideas, Grades K-12:* Twenty-Fourth Yearbook, The Council, Washington, D.C., 1959.

8. NCTM, *The Revolution in School Mathematics:* The Council, Washington, D.C., 1961.

---

* The Complete address of the National Council of Teachers of Mathematics is: 1201 Sixteenth Street, N. W., Washington, D.C.: 20036.

9. National Education Association, *The New Mathematics:* The Association, Washington, D.C., 1964.

10. NCTM, *Research in Mathematics Education:* The Council, Washington, D.C., 1967,

II. *Magazine Articles*

11. ASCD, "Mathematics in the Schools" *Educational Leadership,* Vol. 19, (March, 1962), pp. 353-389.

12. NEA, Department of Audio-Visual Instruction, "The New Mathematics," *Audio-Visual Instruction,* Vol 7, (March 1962), pp. 136-386.

13. Zant, James H., "Using New Media in the Mathematics Classroom", *Educational Technology,* Vol. VI, No. 13, (June 30, 1966), pp. 1-8.

III. *Films:*

14. NCTM, *Mathematics for Tomorrow,* 29 min., sound and color, 1965. Order from Audio-Visual Sound Studio, 1201 Sixteenth Street, N. W., Washington, D.C. 20036.

SECTION II

Ways of Preparing Teachers in a Particular School System to Use New Mathematics

Introduction

Curriculum revision on mathematics at all levels has been going on at a rapid rate all over the country for a number of years. While much of the present curriculum is still in the experimental stage, definite changes relating to both the presentation and the content of mathematics is now evident in school programs and textbooks in use in the schools. Examination of new textbooks published for future use indicate further changes. With the results of experimental projects now in progress it seems probable that such changes will be continuing for some time.

These curriculum changes represent a definite move toward the so-called New Mathematics or Modern Mathematics. It is true that Oklahoma has had a long and successful experience in using experimental textbooks of the SMSG variety and also since 1964 in the use of some of the more modern texts from the present adoption. However, many teachers in the state have not used the experimental texts, nor have they used the more modern texts of the last few years.

The indications are now that many of these teachers, perhaps all of them, will be faced with the task of using books containing the new concepts which they have been able to avoid until now. Hence, this section will offer some suggestions about how teachers may prepare themselves to meet the problems which will almost surely face them in the future.

The Problem

The problem is to help teachers prepare themselves to handle new concepts and topics in modern mathematics with confidence and efficiency. There will be differences in background knowledge and in the preparation needed between the elementary school teacher and the secondary school teacher, but many of the factors will be the same. The problem of the best retraining program for mathematics teachers in the schools is not yet solved and no one program will be recommended at this time. Much experimenting is still going on with this and it is probable that more than one way will be found to do the job effectively. Some different approaches will be listed later in this section.

Arrangements to make possible this new learning for teachers and to inspire them to do it will probably fall to the school administrators. Colleges will be able to provide some assistance through in-service programs and summer courses, but it is not possible for them to meet all of the demands for such programs and to reach all parts of the state. Well qualified high school teachers and supervisors can be of much assistance with elementary school teachers and with their colleagues in the secondary school. However, it will be necessary for them to have some free time for this purpose. Though the teachers will need help, basically it is an individual problem for each of them. The best program will not be successful unless the teacher is vitally concerned and willing to put in a considerable amount of individual study and effort. Perhaps with some of the newer teaching materials, such as, programmed instruction, films, film strips, and improved textbooks, it will be possible for teachers to take a greater part of the responsibility for their own self-improvement.

It is true that the younger teachers, those trained during the last few years in our Oklahoma colleges, are much better prepared to work with these new concepts than those who completed their college work a few years back. All of the colleges have improved their teacher preparation for mathematics teachers during the last few years. This is particularly true for those expecting to teach in the elementary schools. Hence, it is felt that these young people will not have any great difficulty in working with the new concepts now being incorporated into the school textbooks.

It is pointed out that in-service education should be a permanent part of the planning in any school system since obsolescence in the classroom is as real a problem as obsolescence in industry. There is a great fallacy in thinking that the awarding of a degree at the graduate or undergraduate level signals the end of the learning process for the teacher.

The Cambridge Report, *Goals for School Mathematics**, suggests possible curriculum changes for the twenty-five year period ahead. After reading the report one is aware that changes will continue in mathematics both in content and pedagogy. A well planned, long range in-service program is a feasible way of keeping a staff abreast of and participating in these changes.

Nature of the Preparation Needed

Oklahoma has had enough experience in preparing teachers to work with the experimental textbooks at both the elementary school and the secondary school levels to know that what is needed is a knowledge and understanding of the basic principles of mathematics involving the areas which the teachers will be teaching. Such a general statement as this is difficult for one not already familiar with the concepts to visualize and some examples will be given.

Such a list of topics and basic concepts with which an elementary school teacher should be familiar includes the following:

Pre-number Ideas
Sets and Set Operations
Number Versus Numeral
Numeration Systems
Mathematical structure used in developing the algorithms for all operations, that is, closure, commutativity, associativity, identity, inverses and distributivity.
Factors and Primes
Rational Numbers and Operations on Them
Role of Identity Element in Equivalent Fractions
Ratio, Percentage, Decimals
Sentences, Number Line, and Inequalities
Integers and Operations on Them
Points, Lines, Planes, Spaces
Open and Closed Figures in Two and Three Spaces
Congruence and Similarity
Metric Properties of Figures

---

*For a complete listing of references such as this see Bibliography at the end of the Section.

Since elementary algebra is, in fact, a generalization of the real number system, it is also recommended that elementary school teachers have college level study of algebra in addition to the above topics on number systems and their subsystems and basic informal geometry. This also serves the purpose of giving the teacher some acquaintance with subject matter somewhat beyond the level at which they are teaching.

The secondary school teacher should be familiar with the topics listed for the elementary school teacher and in addition the following concepts and subject matter:

The Structure of Algebra, that is, the real number system as a complete ordered field
The Algebra of Polynomials
Systems of Equations
Matrices, Vector Spaces
Vector Geometry
Logic and Deductive Reasoning
Geometry as a Mathematical System
Ruler and Compass Constructions
Non-Euclidean and Other Geometrics
Analytic Geometry
Functions, Including Polynomial, Circular, Exponential and Their Inverses
Limit and Continuity
Infinite Series
Numerical Trigonometry
Probability and Statistics
Differential Calculus
Integral Calculus With Emphasis on the Definite Integral as an Infinite Sum
Applications in all Areas

Reports by the Committee on the Undergraduate Program in Mathematics (CUPM) and the National Council of Teachers of Mathematics (NCTM) on programs of study and detailed course guides recommended for elementary school and secondary school teachers have been made and will be summarized in the Appendix at the end of this Section.

Developing Preparation Programs for Teachers in the Schools

I. *Preliminary Considerations*

A look back at the long lists of topics, the number of semester hours required and a consideration of all of the other things a teacher is expected to do, indicates that programs for further preparing teachers while they are working must be satisfied with less than is considered desirable. One way to reduce, for the time being, the amount of subject matter to be studied is to provide work for teachers at the various levels at which they are currently teaching. That is, teachers in the first three grades have limited use for operations with fractions; teachers of the fifth and sixth grade like to think that someone in the lower grades has taught the basic notions of addition, subtraction, etc. Such an arrangement would, however, make the task of the person directing the study of all of the teachers much more difficult.

The content for the teachers, that is, the knowledge and understanding of the basic concepts of number and geometry, and for the high school teacher, concepts of algebra, synthetic geometry, function and the like, must be acquired if he is to teach a modern program in mathematics with understanding and efficiency.

Experience in working at the in-service level with Oklahoma teachers has indicated that discussion of methods does enter into the development of the mathematical concepts at various levels. Teachers want methods because they have been conditioned to this point of view by their pre-service preparation, but, without a basic understanding of the mathematics involved, premature consideration of methods is likely to undermine the very thing we are trying to do for the children in improving the program in mathematics. There are differences of opinion on both the place and the importance of methods in the teaching of mathematics, but methods of presenting a topic must never take the place of a true understanding of its meaning and significance in the mathematics program.

Programs for In-Service Education in Mathematics have long been a concern of the National Council of Teachers of Mathematics and have proven to be one of the most successful ways of assisting teachers in learning new concepts and in acquiring additional background knowledge in mathematics. This concern has led to an excellent publication: NCTM, *In-Service Education in Elementary Mathematics,* 1967. Though this is aimed primarily at the elementary school teachers, many of the suggestions apply also to secondary teachers.

This pamphlet contains excellent resource materials for both teachers and administrators who are interested in improving their mathematics programs at both the elementary and secondary school levels. There are many descriptions of actual programs in operation in various parts of the country. There is an excellent bibliography and a list of the 42 State Supervisors of Mathematics and those from the other 8 states who can provide information on what is being done in these states. It is recommended that school people in Oklahoma, both teachers and administrators, provide themselves with copies of this report.

Based on suggestions made specifically in this pamphlet, and more generally by mathematics educators, types of in-service programs may be divided into three categories: Directed Long-Term Study, Directed Short-Term Study and Self-Directed Study.

"The ultimate goal of in-service education in mathematics is to assist each teacher to develop the background needed for effective teaching of mathematics in the classroom." In the following paragraphs these three types of In-Service Study Programs will be discussed in detail.

II. *Directed Long-Term Study*

For those teachers who have little recent preparation in college, other in-service programs and/or experience in teaching modern programs in mathematics in the schools, directed long-term study for a semester, an academic year or even longer is the most beneficial type of in-service program for improving their competence as mathematics teachers. These studies are and may be conducted under: 1. the local system's sponsorship, 2. state and local sponsorship, or 3. national, state, and local joint sponsorship. An example of each of these at the elementary level will be presented here and others may be found in the pamphlet mentioned above.

1. Local System Sponsorship

   Example: Greenwood, Delaware Plan

   In Greenwood, Delaware, a *rural district,* approximately ten elementary school teachers and a secondary mathematics teacher attended a two-week summer training session conducted by the state supervisor of mathematics. (Similar programs have been sponsored by local colleges and universities and supported by NSF. They may also be supported by the U. S. Office of Education.) The training session was directed to demonstrating typical presentations with elementary school teachers and familiarizing the prospective leaders with elementary textbooks and programs. The secondary teacher in turn presented 15 weekly two-hour sessions during the year and after school. NDEA Title III funds were used, in part, to employ a secondary school teacher as a state consultant to lead the in-service programs. NCTM, *In-Service Education in Elementary School Mathematics,* p. 13.

2. State and Local Sponsorship

   Example: The State of West Virginia Plan

   The State of West Virginia sponsors a ten week course in mathematics for elementary school personnel that meets for two hours per week. The lessons, which have been prepared by the *State Mathematics Specialist,* provide an overview of the newer thinking in mathematics and background mathematics for the elementary school program. The importance (See Seaton E. Smith, *Modern Mathematics for Elementary School Teachers,* Charleston, W. Va., 1964) of concepts and their application to the total program is stressed. Some attention is given to the use of teaching aids.

   "The extent to which methods become a part of the course depends upon the instructor. The instructors selected have taken the course themselves, as well as having prior mathematics background. Whether secondary school teachers, elementary school supervisors, or key teachers, they have evidenced an interest in elementary school mathematics and an ability to communicate successfully with elementary school teachers.

   "Lessons provide an introduction to systems of numeration, operations on the rational and real number systems and informal geometry and measurement. Each lesson develops the mathematical background and presents a series of related exercises and a bibliography related to the topic. Liberal use of diagrams and charts helps to organize and clarify ideas.

   "Request for the course is made by the county superintendent to the *State Mathematics Specialist.* A small library from the

State Department of Education is loaned to each center for the duration of the course. The salary of the workshop instructor and the cost of materials come from NDEA Title III funds." NCTM, *In-Service Education in Elementary School Mathematics,* p. 21.

3. National, State and Local Sponsorship

Example: The Project Idaho Plan

"The National Council of Teachers of Mathematics supported a pilot leadership training project (called *Project Idaho*) in the State of Idaho. (One of the seven institutes for 275 high school mathematics teachers and supervisors supported by NSF during the summer of 1967 in which the participants return to work with elementary school teachers is a continuation of Project Idaho.) Selected instructors for in-service programs for elementary school teachers were drawn mainly from the group of secondary school mathematics teachers who had attended NSF (summer) Institutes or Academic Year Institutes. A few principals and superintendents were included in the group. The selected candidates were brought together for a two-week orientation workshop designed to acquaint them with the elementary school program in general, with the mathematics of the elementary school, and with the materials available for in-service work with elementary school teachers.

The training program included:

1. A study of elementary school programs developed in demonstration projects, such as, text materials, teachers' manuals, and films.
2. Discussions with special teachers who had successful experiences in teaching mathematics at the elementary level.
3. Setting up specific plans for in-service education of elementary school teachers in their respective school districts.

"During the first year of the program, the twenty-four persons who received training became mathematics consultants for the school districts with responsibility for in-service work with elementary school teachers in the district."

NCTM, *In-Service Education in Elementary School Mathematics,* pp. 33-34.

Though these illustrations involve work with elementary school mathematics teachers, much of the procedure could be applied to up-grading secondary school mathematics teachers.

III. *Directed Short-Term Study*

Short-term studies, from half a day to two weeks have been used rather extensively in Oklahoma, as well as in many other states. They may be classified as orientation conferences or as short-term workshops or institutes. Sponsors include local districts, state agencies, colleges and universities, commercial publishers, local and national associations of mathematics educators. More often than not, these programs are about mathematics as contrasted to mathematics itself.

There is not time enough to discuss adequately any sizable portion of the mathematical concepts involved. While this activity is helpful to mathematics teachers, it is not adequate to enable a teacher to get enough understanding of any topics to carry on in the classroom for any extended length of time.

For example, the State of Georgia has "sponsored mathematics conferences for superintendents and key teachers in each of three congressional districts. At four all-day conferences held during the year presentations and discussions were concerned with topics such as the following:

Mathematics in a Changing World
Objectives of a Modern Program
Books, Materials, and Media
Mathematics In-Service Education
Organizing and Administering a Modern Program
The Role of NDEA Title III Funds in the Purchase of Materials
Content and Method in a Modern Program

"The responsibilities of both the superintendent and the teacher were considered as participants made plans for similar conferences in their school systems." NCTM, *In-Service Education in Elementary School Mathematics,* pp. 9-10.

This is not an exceptional case and one realizes immediately that leaders have learned and can carry back to their schools only certain definite, though quite useful, things *about* a modern program in mathematics for the schools. This does not qualify the leaders or the teachers with whom they work to do "effective teaching of mathematics in the classroom." Such programs are not adequate and will by themselves have little effect in the classroom. The danger is that both administrators and teachers may think they are.

IV. *Self-Directed Study*

Self-study can be a rewarding activity for any teacher and it may be carried on individually or in small groups within the school. It does take a considerable amount of self-discipline and hard work, and there will be times when any teacher will need help on the hard parts. Unless care is taken such self-study may involve only the ideas *about* mathematics discussed above and prove of little value to the teacher in the classroom. This limitation is not necessary, however, since source material does exist in the form which teachers can use on their own.

Source Materials for Preparation Programs

The materials listed here, books, films and magazines, may be used for directed long-term and short-term study and for self-directed study. The quality and ease of understanding varies, of course, but they are listed somewhat in order of usefulness. Administrators and supervisors contemplating programs in mathematics for their teachers would do well to consult with the State Supervisor of Mathematics, college mathematicians and others concerning source material available.

1. *Books*

Books are still the least expensive, the most easily obtained and the most inclusive source for the improvement of the mind.

There are a number of excellent books involving the concepts of mathematics considered useful for and usable by teachers. Some will be listed here and others will be included in the Bibliography at the end of this section.

SMSG, Studies in Mathematics, Vol. IX, *A Brief Course in Mathematics for Elementary School Teachers,* 1963.

Gerald K. Goff and Milton E. Berg, *Basic Mathematics, A Programmed Introduction,* 1968.

Dora McFarland and Eunice M. Lewis, *Introduction to Modern Mathematics for Elementary Teachers,* 1966

Howard F. Fehr and Thomas J. Hill, *Contemporary Mathematics for Elementary Teachers,* 1966.

Yearbooks of National Council of Teachers of Mathematics: *Twenty-third: Insights into Modern Mathematics,* 1957. *Twenty-fourth: The Growth of Mathematical Ideas,* K-12, 1959. *Twenty-seventh: Enrichment Mathematics for the Grades,* 1963. *Twenty-eighth: Enrichment Mathematics for High School,* 1963. *Twenty-Ninth: Topics in Mathematics for Elementary School Teachers,* 1964.

NCTM, *Mathematics for Elementary School Teachers,* (To be used with the NCTM films.)

It is also recommended that the following pamphlets be obtained for their extended bibliographies:

NCTM, *In-Service Education in Elementary School Mathematics.*

CUPM, *Mathematics Text Materials for the Undergraduate Preparation of Elementary School Teachers.*

CUPM, *Course Guides for the Training of Teachers of Junior High and High School Mathematics.*

SMSG, *Study Guides in Mathematics, Algebra, Geometry, Number Theory, Probability and Statistics,* 1962.

William L. Schaaf, *The High School Mathematics Library,* Third Edition, 1967.

Clarence E. Hardgrove, *The Elementary and Junior High School Mathematics Library,* 1960.

Attention is also called to textbooks written during the last few years for elementary school and secondary school students and to the manuals for teachers which go with these books. Much can be learned from these.

2. *Films*

SMSG, Film for use with *A Brief Course in Mathematics for Elementary School Teachers,* 30 thirty-minute films, color, sound, 16 mm.

NCTM, *Films in Mathematics for Elementary School Teachers,* 10 films, 30 min., 16 mm., sound, color. (These films are available at the following schools: University of Oklahoma, East Central State College, Northeastern State College, Northwestern State College and the Muskogee Public Schools.)

Other films, as well as TV kinescopes, are also available, but the cost of all series is prohibitive for even the large city school in Oklahoma. If one or more of the film libraries in the state could be persuaded to purchase them, the rent would probably be reasonable.

3. *Magazines*

The Arithmetic Teacher
The Mathematics Teacher
The Mathematics Student Journal

Preparation Programs for Secondary School Mathematics Teachers

I. *Introduction*

While the preceding discussions deal mostly with the preparation of teachers of elementary school mathematics, much of it, as has been stated, also applies to secondary school teachers of the subject. However, more should be said about preparing the secondary school teacher for his task in the schools.

First, the secondary school teacher of mathematics is more specialized, but at the same time must be able to teach a wider variety of subjects in the field of mathematics. Second, the adequate secondary school teacher of mathematics must devote a full college course to this preparation. It is perhaps theoretically possible to obtain a part of this training through in-service work, but it is not practical, since, for one thing, the number of teachers in a given area, especially outside the large independent school districts, is too small to make possible the organization of extension and in-service classes.

II. *General Characteristics of the Preparation Program*

The total pattern of preparation for teaching in the secondary school should provide general education, subject matter specialization to ensure an adequate background for the position to be filled, and professional education which should assure competence in presenting subject matter to students.

The National Association of State Directors of Teacher Education and Certification (NASDTEC) has issued a pamphlet, *Guidelines for Preparation Programs of Teachers of Secondary Science and Mathematics,* 1962, which discusses some of the problems involved. "A dominant purpose in the development of these *Guidelines* has been to furnish a flexible instrument for State Directors of Teacher Education and Certification in discharging their chief responsibility —to assure the provision of adequately prepared teachers for the public schools." (p. 2). In Oklahoma the legal responsibility for the approval of the preparation and certification of teachers rests with the State Department of Education. Before the NASDTEC report was developed the Oklahoma State Department of Education had adopted "the approved program approach to teacher education and certification." This process places upon each institution the responsibility for developing teacher preparation programs on the basis of State Guidelines which conform to NCATE standards. In fact, Oklahoma State Department of Education Officials and representatives from many of the colleges and universities in the state have served on both the NASDTEC and NCATE committees for developing these standards.

All institutions in Oklahoma which recommend persons to the State Department of Education for teaching certificates at all levels do so on the basis of "approved programs."

III. *Guidelines for Preparation Programs of Teachers of Secondary School Mathematics*

These *Guidelines* are quoted from the NASDTEC-AAAS report mentioned above, *Guidelines for Preparation Programs of Teachers of Secondary School Science and Mathematics,* pp. 23-26.

1. "The program should include a thorough college-level study of the aspects of the subject that are included in the high school curriculum.
2. "The program should take into account the sequential nature of the subject to be taught, and in particular should provide the prospective teacher with an understanding of the aspects of the subject which his students will meet in subsequent courses.
3. "The program should include a major in the subject to be taught, with courses chosen for their relevance to the high school curriculum.
4. "The program should include sufficient preparation for the later pursuit of graduate work in mathematics.
5. "A fifth-year program should emphasize courses in subject to be taught.
6. "The program should include preparation in the methods especially appropriate to the subject to be taught.
7. "The program should include work in areas related to the subject to be taught.
8. "The program should take into account the recommendations for curriculum improvement currently being made by various national groups."

Specifics are included within the section of these guidelines, but these are perhaps stated more specifically in the CUPM Recommendations quoted in part in the Appendix at the end of this section.

Summary of Developing Preparation Programs for Teachers in the Schools

The following suggestions may prove useful:

I. The content of the programs for teachers of mathematics must be primarily in the concepts, understanding and the structure of mathematics. This will vary with the level at which the teacher works.

II. Methods are also important, but they must be based on a true understanding of the basic mathematical concepts involved. There are differences of opinion on both the place and the importance of methods in the teaching of mathematics.

III. The teacher in the schools must be willing to spend the necessary time and effort to learn and understand the basic concepts of mathematics important to her level of instruction.

IV. Under present conditions the willingness to make this effort must be inspired by and necessary details must be carried out by the administrative staff, superintendents, principals and supervisors. The leadership of the State Supervisor of Mathematics is very beneficial in this area.

V. There are many ways in which the preparation of teachers can be updated and improved. Three categories have been suggested.
   1. Directed Long-Term Study
   2. Directed Short-Term Study
   3. Self-Directed Study

VI. Source material is available for this study and self-study, though it is better at some levels than at others.

VII. Basically self-improvement must be an individual effort put forth by the teacher.

VIII. It is known from previous experience with teachers in programs involving the use of new concepts in mathematics that much is gained from the experience of teaching such courses in the classroom. The first year is difficult and conscientious teachers tend to feel that they do not do a good job. However, the second year is much better and with some experience they become enthusiastic about the program. One reason for this is the response received from the pupils in the class; often for the first time they are able to understand mathematics and it makes sense to them. Team teaching can be very helpful to those involved.

Recommendations

The following recommendations are made to teachers and administrators:

I. Select the best textbook available, keeping in mind the needs of the school and the community, the students who will be taking the courses in mathematics, and the teachers of the system.

II. The superintendent and his administrative staff are the logical ones in the school system to take the responsibility of getting the teachers prepared to teach the material in the curriculum and the textbooks. The teachers, however, must approach this as an individual self-improvement effort, but the State Supervisor of Mathematics can be helpful.

III. Provide some sort of program to assist the teachers in this effort.

This program may be organized as:

1. Large groups or classes taught by a supervisor, another teacher or, if possible, a college mathematics teacher. There is a possibility that some instruction will be available by educational television and movies. School systems can work cooperatively in this respect to get a group large enough.
2. Small study groups, by schools or in smaller units. A difficulty may be to obtain adequate leadership and help for these small groups, but self-directed study can be very valuable.

3. If teachers are willing to assume the responsibility, we are convinced that many of them can make a considerable improvement. Many gain a modern know-how as well as revitalizing interest and enthusiasm in teaching mathematics.

IV. Teachers should be encouraged to try the new approaches and the new concepts, as suggested in the selected textbooks, consistently in class. They should study and teach the content to the best of their ability. If this is done in addition to the suggestions made in No. 111 above, it will not only motivate the proper effort on the work of No. III, but the teacher will be able to do a much better job of teaching her class the following year.

V. School administrators should budget money on a continued basis for in-service opportunities for their teachers. This could be on a rotating system with mathematics in-service every three or four years. Money should also be budgeted for teachers to attend regional and national meetings of the National Council of Teachers of Mathematics.

VI. Memberships should be encouraged by the administration in the professional organizations of O.C.T.M. and N.C.T.M. as well as in O.E.A. and N.E.A.

Bibliography

For Elementary School Teachers

I. *Books*

1. CUPM, *Recommendations for Training of Teachers of Mathematics,* Berkeley, Calif.; 94701, CUPM Central Office, P. O. Box 1024, Free.
2. CUPM. *Course Guides for the Training of Teachers of Elementary School Mathematics,* Berkeley, Calif.: 94701, CUPM Central Office, P. O. Box 1024, Free.
3. CUPM. *Mathematics Text Materials for the Undergraduate Preparation of Elementary School Teachers,* Berkeley, Calif.: 94701, CUPM Central Office, P. O. Box 1024, Free.
4. Fehr, Howard F. and Thomas J. Hill. *Contemporary Mathematics for Elementary Teachers,* Boston, Mass.: 02116, D. C. Heath and Company, 285 Columbus Ave., 1966.
5. Goff, Gerald K. and Milton E. Berg, *Basic Mathematics, A Programmed Introduction,* New York: 10001 Appleton-Century-Croft, 34 West 33rd Street, 1968.
6. McFarland, Dora and Eunice M. Lewis, *Introduction to Modern Mathematics for Elementary Teachers,* Boston, Mass.: 02116, D. C. Heath and Co., 285 Columbus Ave., 1966.
7. NASDTEC-AAAS, *Guidelines for Preparation Programs of Teachers of Secondary School Science and Mathematics,* Washington, D.C. 20036, NASDTEC-AAAS Studies, 1515 Massachusetts Avenue, N. W., Free.
8. NCTM, *In-Service Education in Elementary School Mathematics,* Washington, D.C. 20036, NCTM, 1201 Sixteenth Street, N.W., 1967.

9. NCTM. Yearbooks of National Council of Teachers of Mathematics

   Twenty-third: Insights into Modern Mathematics, 1957.
   Twenty-fourth: The Growth of Mathematical Ideas, K-12, 1959.
   Twenty-seventh: Enrichment Mathematics for the Grades, 1963.
   Twenty-eighth: Enrichment Mathematics for High School, 1963.
   Twenty-ninth: Topics in Mathematics for Elementary School Teachers, 1964.

   (All of these may be ordered from Washington, D.C., 20036, NCTM, 1201 Sixteenth Street, N.W.)

10. NCTM. *Mathematics for Elementary School Teachers,* (To be used with the NCTM films) NCTM, 1965.

11. Ohmer, Merlin W., Clayton V. Aucoin and Marion J. Cortez. *Elementary Contemporary Mathematics,* New York: Blaisdell Publishing Co., 1964.

12. Peterson, John A. and Joseph Hashisaki. *Theory of Arithmetic,* New York: 10016, John Wiley and Sons, 440 Park Ave. South, 1963.

13. SMSG. Studies in Mathematics, Volume IX, *A Brief Course in Mathematics for Elementary School Teachers,* Pasadena, Calif.: 91105, A. C. Vroman, Inc., 367 S. Pasadena Ave., 1963.

14. Smith, Seaton E. *Modern Mathematics for Elementary School Teachers,* Charleston, W. Va., State Department of Education, Division of Mathematics, 1964.

15. Ward, Morgan and Clarence E. Hardgrove. *Modern Elementary Mathematics,* Reading, Mass.: Addison-Wesley Publishing Co., 1964.

II. *Films*

16. NCTM. *Films in Mathematics for Elementary School Teachers.* 10 films, 30 min., 16 mm., sound, color. (Distributed by United World Films, Inc., 221 Park Avenue, South, New York: 10003, 1965.)

17. SMSG. Film for use with *A Brief Course in Mathematics for Elementary School Teachers,* 30 thirty minute films, color, sound. 16 mm. Available from New York: 10022, Modern Learning Aids, 3 East 54th Street.

III. *Magazines*

18. NCTM. *The Arithmetic Teacher,* Washington, D.C.: 20036, NCTM, 1201 Sixteenth Street, N. W.

For Secondary School Teachers

I. *Books*

19. Allendoerfer and Oakley, *Principles of Mathematics,* New York: 10022, McGraw-Hill Book Co., 330 W. 42nd Street.

20. Birkhoff and Beatley. *Basic Geometry,* New York: Chelsea Publishing Co., 50 East Fordham Road.

21. Cambridge Conference on School Mathematics. *Goals for School Mathematics,* Boston, Mass. 02107: Houghton Mifflin Co., 110 Tremont Street, 1963.

22. CUPM. *Course Guides for the Training of Teachers of Junior High and High School Mathematics,* Berkeley, Calif.: 94701, CUPM Central Office, P. O. Box 1024, 1965, 35 pp. Free.

23. Haag, V. H. *Structure of Elementary Algebra,* Studies in Mathematics, Vol. III, (SMSG), Pasadena, Calif.: 91105, A. C. Vroman Co., Inc., 367 S. Pasadena Ave., 1960.

24. Haag, V. H. *Structure of Algebra,* Reading, Mass.: Addison-Wesley Publishing Co., 1964.

25. Kemeny, Mirkil, Snell and Thompson. *Finite Mathematical Structures,* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1959.

26. Moise, Edwin E. *Elementary Geometry From an Advanced Point of View,* Reading, Mass.: Addison-Wesley Publishing Co., 1963.

27. Prenowitz and Jordan. *Basic Concepts of Geometry,* Boston, Mass.: Ginn and Company, 1961.

28. SMSG. *Introduction to Matrix Algebra,* Pasadena, Calif. 91105: A. C. Vroman, Inc., 367 S. Pasadena Ave., 1960.

29. SMSG. Studies in Mathematics (SM-14) *Introduction to Number Systems,* Pasadena, Calif.: A. C. Vroman Co., Inc., 1964.

30. SMSG. *Study Guides in Mathematics,* Pasadena, Calif.: A. C. Vroman Co., Inc., $.50 (Includes Algebra, Calculus, Digital Computing and Related Mathematics, Geometry, Intermediate Mathematics, Number Theory, Probability and Statistics. For teachers grades 9-12.)

II. *Magazines*

31. MAA. *The Mathematics Magazine,* Buffalo, N. Y.: 14214, Mathematical Association of America, State University of New York.

32. NCTM. *The Mathematics Teacher,* Washington, D.C.: 20036, NCTM, 1201 Sixteenth Street N.W.

33. NCTM. *The Mathematics Student Journal,* Washington, D.C.: 20036, NCTM, 1201 Sixteenth Street, N. W.

APPENDIX

The CUPM Recommendations for Training Mathematics Teachers

The Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America (MAA) has made an extensive study of the nature of mathematics needed for teachers in the elementary school, as well as for teachers at the secondary and college levels. Conferences involving teachers, administrators, college mathematicians and mathematics educators were held in many states over the country, with the first one here in Oklahoma. At these conferences tentative proposals were discussed and the possibilities of inaugurating such programs in colleges for pre-service preparation and in the schools for in-service preparation were thoroughly explored.

The CUPM suggestions and recommendations have had wide effect in Oklahoma and in the colleges of the state which prepare teachers. The recommendations have also widely affected the in-service and extension classes, workshops and institutes held for elementary and secondary school teachers. For these reasons many of the teachers in Oklahoma at all levels have been able to improve themselves and as a result of this are able to teach the new mathematics effectively in the schools.

These recommendations of CUPM are quoted here in some detail because of the influence they have had in the area of mathematics education and for reference purposes. They have been mentioned several times earlier in this section and their influence is discernable in much of the material which has been produced for teachers during the last few years. The recommendations can serve a dual purpose: that of informing teachers in service of the knowledge needed to keep pace in the profession; and that of serving as guidelines for the employment of new teachers in a given school system.

Recommendations for Level I*

(Teachers of Elementary School Mathematics)

"As a prerequisite for the college training of elementary school teachers, we recommend at least two years of college preparatory mathematics, consisting of a year of algebra and a year of geometry, or the same material in integrated courses. It must also be assured that these teachers are competent in the basic techniques of arithmetic. The exact length of the training program will depend on the strength of their preparation. For their college training we recommend the equivalent of the following:

(a) A course or a two-course sequence devoted to the structure of the real number system and its subsystems (Course 1)

(b) A course devoted to the basic concepts of Algebra (Course 2)

(c) A course in informal geometry (Course 3)

"The material in these courses might, in a sense, duplicate material studied in high school by the prospective teacher, but we hope that this material will be covered again, this time from a more sophisticated college-level point of view.

"Whether the material suggested in (a) above can be covered in one or two courses will clearly depend upon the previous preparation of the student.

"We strongly recommend that at least 20 per cent of the Level I teachers in each school have a stronger preparation in mathematics, comparable to Level II preparation, but not necessarily including calculus. Such teachers could strengthen the elementary program by their very presence within the school faculty. This additional preparation is certainly required for elementary teachers who are called upon to teach an introduction to algebra or geometry." (In an eight year elementary school, for example.)

---

*See CUPM "Training of Elementary-School Mathematics Teachers" The Arithmetic Teacher, Vol. No. 8, (Dec. 1960) pp. 421-425, or CUPM Recommendations for the Training of Teachers of Mathematics, CUPM Central Office, P. O. Box 1024, Berkeley, California 94701, pp. 11-13. Free.

Course Descriptions

"We list below sample courses that might be used to fulfill the minimum requirements for Level I . . . The brief descriptions are included to clarify the meanings of the course titles, but are not intended as course syllabi for actual courses. It must be recognized that there are other equally good ways of combining various recommended topics, and colleges should be encouraged to work out detailed curricula to suit their own tastes and local conditions. However, the committee hopes that these very brief descriptions will help in indicating the types of courses desirable and the level of advancement.

For Level I

"1. *Algebraic structure of the number system* (2-course sequence). This is a study of the numbers used in elementary school—whole numbers, common fractions, decimal fractions, irrational numbers.

"Emphasis should be on the basic concepts and techniques: properties of addition, multiplication, inverses, systems of numeration, and the number line. The techniques for computations with numbers should be derived from the properties and structure of the number system, and some attention should be paid to approximation. Some elementary number theory, including prime numbers, properties of even and odd numbers, and some arithmetic with congruences should be included.*

"2. *Algebra.* Basic ideas and structure of algebra, including equations, inequalities, positive and negative numbers, absolute value, graphing of truth sets of equations and inequalities, examples of other algebraic systems—definitely including finite ones—to emphasize the structure of algebra as well as simple concepts and language of sets.*

"3. *Intuitive foundations of geometry.* A study of space, plane, and line as sets of points, considering separation properties and simple closed curves; the triangle, rectangle, circle, sphere, and other figures in the plane and space considered as sets of points with their properties developed intuitively; the concept of deductive theory based on the properties that have been identified in intuitive development; concepts of measurement in the plane and space, angle measurement, measurement of the circle, volumes of familiar solids; treatment of coordinate geometry through graphs of simple equations."*

This rather long description of a suggested program for preparing prospective elementary school teachers while still in college is given here because it also gives a suggestion about a suitable program of preparation for teachers in service who had no opportunity or requirement while they were in college. Some adjustments will have to be made, but the beginning part of in-service training should fall within the area of number and geometry. Some books and other materials which cover these topics have already been listed in the first part of this section.

For the Teacher of Secondary School Mathematics

The needs of the secondary school teacher are more varied and spread over a wider range; the junior high school teacher needs different content and perhaps less of it than those who teach the upper three grades. However, as with the elementary school teacher, they will need a basic knowledge and understanding of the principles of mathematics adjusted to their level of teaching.

---

*For a list of suggested topics see: CUPM, Course Guides for the Training of Teachers of Elementary School Mathematics.

The CUPM recommendations lists four levels, II-IV inclusive, with V for college mathematics teachers. The recommendations for II, III, and IV are as follows:*

"Recommendations for Level II (Teachers of the elements of algebra and geometry)

"Prospective teachers should enter this program ready for a mathematics course at the level of a beginning course in analytic geometry and calculus (requiring a minimum of three years in college preparatory mathematics). It is recognized that many students will have to correct high school deficiencies in college. (However, such courses as trigonometry and college algebra should not count toward the fulfillment of minimum requirements at the college level.) Their college mathematics should then include:

"(a) Three courses in elementary analysis (analytic geometry and calculus) (including or pre-supposing the fundamentals of analytic geometry). (See course-sequence 4.)

"This introduction to analysis should stress basic concepts. However, prospective teachers should be qualified to take more advanced mathematics courses requiring a year of the calculus, and hence calculus courses especially designed for teachers are normally not desirable.

"(b) Four other courses: a course in abstract algebra, a course in geometry, a course in probability from a set-theoretic point of view, and one elective. One of these courses should contain an introduction to the language of logic and sets. (See course sequences 5-7.)"

"Recommendations for Level III. (Teachers of high school mathematics.)

"Prospective teachers of mathematics beyond the elements of algebra and geometry should complete a major in mathematics and a minor in some field in which a substantial amount of mathematics is used. The latter should be selected from areas in the physical sciences, the biological sciences, and the social studies, but the minor should in each case be pursued to the extent that the student will have encountered substantial applications of mathematics.

"The major in mathematics should include, in addition to the work listed under Level II, at least an additional course in each algebra, geometry, and probability, and one elective.

"Thus, the minimum requirements for high school mathematics teachers should consist of the following: (The requirements for Level II Preparation have been included in this list.)

"(a) Three courses in analysis. (See course-sequence 4.)
(b) Two courses in abstract algebra. (See course-sequence 5.)
(c) Two courses in geometry beyond analytic geometry. (See course-sequence 6.)

---

*See "Recommendations of MAA for the Training of Teachers of Mathematics", CUPM Committee, American Mathematical Monthly, Vol. 67, No. 10 (Dec. 1960).

(d) Two courses in probability and statistics. (See course-sequence 7.)

(e) Two upper-class elective courses, for example, introduction to real variables, number theory, topology, history of mathematics, or numerical analysis (including the use of high-speed computing machines).

"One of these courses should contain an introduction to the language of logic and sets, which can be used in a variety of courses."

"Recommendations for Level IV (Teachers of the elements of calculus, linear algebra, probability, etc., these teachers are in some high schools, more often they are in junior college).

"On this level we recommend a master's degree with at least two-thirds of the courses being in mathematics, for which an undergraduate program at least as strong as Level III training is a prerequisite. A teacher who has accomplished the recommendations for Level III should use the additional mathematics courses to acquire greater mathematical breadth.

"It is important that universities have graduate programs available which can be entered with Level III preparation, recognizing that these students substitute greater breadth for lack of depth in analysis as compared with an ordinary B.A. in Mathematics. In other respects, graduate schools should have great freedom in designing the M.A. program for teachers."

These programs in mathematics for secondary school teachers are clear enough to those teachers who have gone through them and to college mathematics teachers over the state. It is not felt necessary at this point to quote the course descriptions given by CUPM, since it is not possible to carry on such courses by extension, in-service programs or summer institutes. If a teacher has not had them then it will be necessary to return to the college or university for this purpose.

Programs for secondary school mathematics teachers are very well developed in Oklahoma. The Oklahoma State University meets Level III requirements (31-33 semester hours of mathematics beginning with analytic geometry and calculus with the other specific courses required) for all students recommended for a Standard Teaching certificate in mathematics. The University of Oklahoma meets the semester hour requirement, but does not require a minimum of six semester hours in algebra, geometry and statistics. Other colleges and universities still allow credit for a two-course sequence (6 semester hours) in trigonometry and college algebra and it is not possible for students in these schools to satisfy a part of the electives suggested by CUPM. Many of our mathematics teachers have participated in NSF Academic Year Institutes, Summer Institutes and have qualifications through Level IV.

It is possible that some of the teachers now in service can through individual study and what help is available through colleagues, college professors and the like, improve their own competence in this field. As mentioned CUPM has course descriptions of all suggested courses. They also have a report "Course Guides for the Training of Teachers of Junior High and High School Mathematics," June 1961, which gives an outline and references to be used for each section throughout each course. These may be used for individual study.

CUPM has this to say further about adequate preparation for elementary and secondary school teachers of mathematics.

"The above recommendations (those quoted in this section) have dealt in detail with the subject-matter training of mathematics teachers. There are many other facets to the education of scholarly, vigorous, and enthusiastic persons to whom we wish to entrust the education of our youth. One of these merits special attention by us. Effective mathematics teachers must be familiar with such items as:

(A) The objectives and content of the many proposals for change in our curriculum and texts.

(B) The techniques, relative merits, and roles of such teaching procedures as the inductive and deductive approaches to new ideas.

(C) The literature of mathematics and its teaching.

(D) The underlying ideas of elementary mathematics and the manner in which they may provide a rational basis for teaching, unless taken care of by mathematics courses especially designed for teachers.

(E) The chief applications which have given rise to various mathematical subjects. These applications will depend upon the level of mathematics to be taught and are an essential part of the equipment of all mathematics teachers.

"Such topics are properly taught in so-called 'methods' courses. We would like to stress that adequate teaching of these can be done only by persons who are well informed both as to the basic mathematical concepts and as to the nature of American Public Schools, and as to the concepts, problems, and literature of mathematics education. In particular, we do not feel that this can be done effectively at either the elementary or secondary level in the context of 'general' methods courses, or by persons who have not had at least the training of Level IV.

Training Supervisors

"There is a great need for providing adequately trained supervisors of mathematics, grades K-12, for our public schools. At present, administrators find no ready supply of such individuals and, hence, are through necessity making appointments which are highly questionable, if not indefensible. For this reason, it is urgent to develop a program for supervisors and to seek adequate support for those individuals who have the desired qualities for supervision and the ability to benefit from advanced training. Such training would prepare the 'leaders of teachers' in the local system (A) to make sound judgments concerning mathematics programs for the schools, (B) to understand thoroughly the recommendations made by the national committees, and (C) to enable schools to better articulate school mathematics with college mathematics.

"Prerequisite to this program should be a regular master's degree in mathematics or a master's degree given as a result of preparation in an Academic Year Institute. The program should consist of additional graduate courses in abstract algebra, analysis, and geometry, with courses selected from logic, statistics, theory of numbers, philosophy of education, history of education, history of mathematics, seminar courses on the program of the elementary school and secondary school mathematics, and additional elective courses in algebra, analysis, or geometry to provide some degree of concentration

"The committee feels [illegible] action must be taken to fill the need for supervisory personnel, and we recommend such action to the appropriate authorities."

SECTION III

Initiating a Modern Program in Mathematics in a Particular School System

Introduction

It is the point of view of this committee that mathematics taught with an emphasis on the fundamental concepts and understanding will give the students in our schools a more basic and useful knowledge of this important area of our culture. Moreover, experience in Oklahoma Schools has shown us that children can learn these concepts and that they are much more interested in doing so than they are in following the rote memorization type of teaching used in the past. Textbooks, on which the curriculum of the schools largely depend, whether we like it or not, have been changing rapidly during the past few years. There are positive indications that more drastic changes will be made in the future. For example, the textbooks in mathematics which were adopted in October, 1967, probably contain more of the new mathematics than has appeared in previous adoptions. A school system should consider carefully the initiation of the best possible program in mathematics and adopt the best textbooks available for their community needs. As stated above, it is the opinion of this committee that the mathematical content and the approach to teaching and learning mathematics should be modern in character. This means, the content should involve the fundamental concepts of mathematics and it should be taught from the standpoint of discovery and understanding. Points which must be considered, of course, are:

1. The types of students in the school, their cultural, social and intellectual background.
2. The future needs of the students for mathematics, if this can be determined.
3. The readiness of the community for change in the mathematics curriculum.
4. The adequacy of preparation of the mathematics staff (See section II).
5. The receptiveness of the mathematics staff to a change in emphasis of the mathematics program.
6. The willingness and ability of the school system to provide and carry through in-service training that may be needed to initiate a modern program.

References:

Edwina Deans, *Elementary School Mathematics: New Directions*
National Council of Teachers of Mathematics, *The Growth of Mathematical Ideas, Grades K-12*
National Education Association, *The New Mathematics*
National Council of Teachers of Mathematics, *Mathematics for Tomorrow, film,* 29 minutes, sound and color.

Mathematics Programs for Different Types of Students

The type of community will determine, to some extent, the need for mathematics by the pupils in the schools. However, the high mobility of our population must also be considered; it is not likely that a large percent of students in school today will be spending their adult years in the same type of community in which they now live. It is also well known that any community in Oklahoma may have students who can do well in mathematics and who can profit greatly from the study of it. Actually we know that in almost any school there are groups of students who may be classified as talented, average, and low achievers. What the school can do for these groups of students will depend on its resources; that is, money, staff, professional advice and various types of teaching aids.

It is often recommended that a school system offer three levels of instruction, one for the talented students, one for the average students and one for low achievers. This will probably be useful, if the school is large enough and has sufficient resources to do so. However, some schools cannot offer three levels of mathematics or even two. To get the best from the students and the best for them, some definite attention must be given to the individual student at his own level. In schools able to offer only one class to accommodate all students, extreme care must be exercised so that different levels of difficulty and rigor may be obtained by the use of exercises in the text. For example, superior students may be assigned some of the same exercises as the average students plus some that are more difficult and challenging. These problems should be included and identified in the textbook. (See Section I.)

References:

AASA, ASCD, NASSP and NCTM, *Administrative Responsibilities for Improving Mathematics Programs.*
NCTM, *Research in Mathematics Education.*
NCTM, *Mathematics for Tomorrow,* film, 29 minutes, sound and color.

For the talented and the average student it is recommended that the mathematics program involve the modern ideas which have been introduced into the mathematics curriculum in the past ten years. Many books are available in which this material is written to be read and studied by the students. Much of it appears in the adopted textbooks. Talented students must be given special attention and teachers will have to spend considerable time assembling material suitable for their development. (See Section V.) Handling courses for these students, both the talented ones and those classified as average, will present no great problem if the teachers are well trained. (For more details see Section II on preparing teachers.) If the teachers are not qualified something must be done to provide help for them, or the students will suffer.

For the low achievers the problem is much more difficult. Traditionally we have given these students more exercises of the fundamental drill sort, that is, more addition problems, more work in fractions and the like. Since they have a long experience of failure on exercises involving these skills, it is not surprising that this effort on the part of the schools has been unsuccessful and they fall further behind. The problem requires much experimentation and work before we will be able to say definitely what can be done.

Federal support is now available through ESEA Title I for disadvantaged children and through Title III for innovations in curricula materials. Some experimental work is already underway in the Oklahoma

City Public Schools and in the Muskogee Public Schools. At present both of these schools are experimenting with students at the secondary school level using Title I funds. There may be others in Oklahoma involved in this type of work and there are a number of projects in other states. We urge schools with available staff to investigate these possibilities. This problem is discussed more completely in Section IV of this book. Definite references are made to projects underway, noting procedures for development and listing materials which are available.

Resources for the Administrator in Developing a Modern Program in Mathematics

Certain resources are available to school administrators interested in improving their mathematics programs. Some of these are: professional leadership in the State Department of Education, in colleges, in the schools and in the various professional organizations over the country. A number of these are in the form of books, pamphlets and articles in the magazines. Many are listed in the Bibliography at the end of this Section. These resources are discussed more specifically below:

References:

AASA, ASCD, NASSP and NCTM, *Administrative Responsibilities for Improving Mathematics Programs.*
ASCD, *Commission on Current Curriculum Developments*
Matthew B. Miles, Editor, *Innovations in Education,* Chapter 14.
NCTM, *In-service Education in Elementary School Mathematics.*

I. *The State Supervisor of Mathematics*

A competent person within the State Department of Education is considered a most important resource in helping to develop a quality program in mathematics in the school systems.

Some of the activities of a State Supervisor of Mathematics in Oklahome are to:

1. Work with teachers and administrators in an advisory capacity for the improvement of their mathematics programs.
2. Work with other officials within the State Department of Education to help provide a coordinated program of education for all of the children in the state.
3. Cooperate with colleges and their mathematics staffs to provide the best possible preparation in mathematics for prospective teachers K-12 and to prepare teachers already in service to teach mathematics effectively in the schools.
4. Initiate and provide leadership for a continuous evaluation of the State Department of Education mathematics requirements for graduation from high school. This includes the possibility of different requirements for different types of programs to be recorded on transcripts.
5. Organize and administer area and state level conferences and workshops for mathematics teachers.
6. Provide teachers and administrators, through bulletins and other means, information on various phases of mathematics education;

such as, experimentation under way, new learning materials and devices, use of new media for mathematics instruction and the like.

7. Collect and disseminate information of Federal support of activities for the improvement of mathematics teaching. Give advice on how projects may be initiated, organized and operated.

References:

AASA, ASCD, NASSP and NCTM, *Administrative Responsibilities for Improving Mathematics Programs*

Cambridge Conference on School Mathematics, *Goals for School Mathematics.*

NCTM, *The Revolution in School Mathematics.*

NCTM, *The Supervisor of Mathematics: His Role in the Improvement of Mathematics Instruction.*

Another suggested activity for the State Supervisor of Mathematics in cooperation with the State Supervisors of other special areas is a joint effort for curriculum improvement under the direction of the Director of Instruction in the State Department of Education. With this type of organization there would be available to the school administration a team of experts who could consider the total problem of organizing the school to best serve the needs of the community. It is seldom true that a particular school needs or desires improvement in a single area. Having several different specialists available to the school would make it possible for the superintendent to unify his efforts for school improvement, but it would also be an advantage to the separate areas, since, in most cases, their efforts can supplement each other and strengthen the entire program.

II. *Local Supervisors and/or Well Qualified Teachers*

The teachers and supervisors (if there are any) of a school are the most important local resources in initiating and carrying out a quality program in mathematics in a particular school system. The administration does, of course, select the best possible staff for the schools, but are usually limited by financial support and the quality of teachers available. If the school administrator is committed to a quality program in the school, he must make a further selection from the staff he has been able to assemble of those persons qualified, able and willing to assume leadership in carrying out this task. This will involve, in most cases, further preparation for some members of the teaching and administrative staff; this is primarily the responsibility of the superintendent and his staff. Suggestions for doing this have been discussed in Section II.

An important point to consider in the use of teachers and other personnel is their time, attitude and enthusiasm. Any innovation introduced into a school system will take time for the administrator and the teacher; furthermore they must want to do it and have some enthusiasm for the task. If the teachers are poorly qualified to teach mathematics from a modern point of view, then additional training must be provided for them. At best this will involve some leadership, such as, a college mathematician who is cognizant of the content of mathematics and teaching problems in the schools. Since college professors are not often available, other persons who could be used by the schools are those secondary teachers of mathematics who have attended NSF Institutes for high school mathematics teachers, but these need to be carefully selected. Those who have attended an NSF Academic Year Institute are often especially well qualified and should be used more extensively than is now being done in the schools.

The National Science Foundation has been offering special summer training in a few Summer Institutes on Mathematics for elementary school personnel (teachers, supervisors and principals) to help them improve their knowledge of content and methods of teaching in the schools.

Example:

For several summers the National Science Foundation has sponsored Summer Conferences for Elementary School Mathematics. The participants are well qualified secondary school mathematics teachers and supervisors who returned to their home school systems and served as leaders for in-service instruction of elementary school personnel during the academic year. These summer institutes have been held in Oklahoma and others will probably be located here in the future.

Such institute programs and others organized by the colleges and universities, the State Department of Education and the Oklahoma Council of Teachers of Mathematics have been going on in Oklahoma for a number of years. The colleges and universities and the State Department of Education have also increased and improved their certification requirements for teachers. Many teachers who qualified for certificates ten or more years ago and who have not been able to attend special programs need further help. It may be necessary in some schools to develop for the teachers a program involving self teaching devices, such as programmed instruction, or self study in a more or less outline form. All of these will involve a considerable teacher self-discipline in a particular school with much individual effort on the part of the teacher. The teacher time involved is an important factor and it is recommended that administrators consider the possibility of releasing teachers one hour one day per week for in-service work in subject matter areas. It is often difficult to get teachers enthusiastic about self-improvement projects if they are expected to use their own time entirely and consistently for this purpose.

In *summary:* to make the most effective use of local personnel in improving mathematics in the schools, the administrator should:

1. After selecting the best staff local limitations will permit, make a further selection from this group to serve as leaders in changing the school's program in mathematics.
2. Consider the time, attitude and enthusiasm of the staff for a change in the mathematics program of the school.
3. Provide some type of in-service program for those who need a broader and more basic knowledge of mathematics and how to teach it at the various levels in the schools. For this some help may be available from the State Supervisor of Mathematics, college mathematicians, the Oklahoma Council of Teachers of Mathematics and/or local school personnel; it may be necessary in some situations to make use of programmed instruction materials or some other new media which lend themselves to self-study on the part of the teacher.

III. *Mathematics Consultants From State Colleges and Universities*

College mathematicians who are cognizant of the content and methods of presentation of mathematics prevalent in the schools and who have the ability to relate this content to classroom practice can be valuable resource persons. While under present conditions in the colleges they ordinarily will not have time to offer courses for teachers, they will be able to advise local schools on the kind of course work needed, outline courses and assist in selecting teachers competent to teach the courses.

IV. *Instructional Materials for the Schools*

Such materials are, of course, necessary for the development of a good program in mathematics for the schools. The textbook is of primary importance. Other learning materials; such as: library books, suitable teaching aids for the class level and the like, are also important. It may be possible in the near future to take advantage of television lessons designed for a particular level of instruction or to procure similar types of teaching materials suitable for movie projectors or closed circuit television. Books or teaching machines using the programmed instruction techniques have considerable potential, especially for the slow learners, who need more emphasis on the material presented, and for the rapid learners, who need new and different topics to work on.

References:

Edwina Deans, *Elementary School Mathematics: New Directions*
Cambridge Conference on School Mathematics, *Goals for School Mathematics*
NCTM, *The Growth of Mathematical Ideas, Grades K-12.*
NCTM, *Topics in Mathematics for Elementary School Teachers*

Careful precautions must be taken, however, when thinking of using television instruction, movies, programmed instruction and other teaching devices in what may be called mass or individual instruction. This means any type of instruction where the teacher is not in control of what the student is doing and what objectives are set up for the instructional unit. Two things are important in these situations. These are: first, the objectives of the instructional unit and second, what the students actually learn from the instruction given. The behavioral objectives and other kinds of objectives of any media learning project should be clearly stated and known to the teacher or other persons who select the television lessons, movie film or programmed instruction unit. Otherwise, there is no way of knowing whether this particular piece of instructional material is suitable for the particular class or level of instruction. Second, evidence should be furnished to indicate that students of the particular level and ability will indeed learn (have their behavior changed) by viewing the television lessons, looking at the film, working through the programmed instruction unit, etc. Anything less than this is not enough. If the students do not learn, then the time of thirty, fifty, or one hundred students has been wasted!

Teaching materials using these various media can be developed on the principles of clearly stated objectives and assurance of learning them, but the task is tedious and time consuming. There is indeed no evidence that any large proportion of the teaching materials now available have been produced according to these principles.

Reference:

James H. Zant "Using New Media in the Mathematics Classroom," *Educational Technology,* Vol. VI, No. 13, (June 30, 1966), pp. 1-8.

It is true that careful planning for the use of all the teaching materials by any good teacher involves, among other things, clearly stated objectives and some way of finding whether the children learn them. Teachers usually know what the objectives are, though they are seldom written down by the teacher and textbooks certainly *do not* state them explicitly. Good teachers, when they are in control of the classroom, have ways of discovering whether or not the students are learning. If they are not, then the subject matter is presented again, often in another way, until it is learned. The point here is that in the use of mass media for teaching, where the material to which the student is exposed will likely be controlled by a centralized computer, the teacher will no longer have control of the classroom and extra precautions will be necessary.

Reference:

NCTM, *Research in Mathematics Education*

If one doubts that the classroom of the future and the content of the curriculum may well be so controlled, it is only necessary to look at the mergers now taking place between mass communication and machine big business and the time-honored producers of textbooks and teaching materials in this country. Examples are: IBM and SRA. The Rayethon Company and D. C. Heath, RCA and Random House. The schools have become big business and big business is moving in on the market!

V. *Professional Organizations, Their Research and Publications*

The contributions of these organizations to education in the schools are well known to administrators and many teachers. References have been made to a number of their publications throughout the discussions in this section and they are listed in the Bibliography at the end of the section; there are others, of course.

In addition to the professional organization primarily for administrators there are those designed primarily for mathematics teachers. The National Council of Teachers of Mathematics (NCTM) membership includes teachers of mathematics at both the elementary school and the secondary school levels. It publishes two journals, *The Arithmetic Teacher* and *The Mathematics Teacher*. In addition it has many other publications which include Yearbooks, the *NCTM Newsletter, The Mathematics Student Journal* and numerous supplementary publications giving information on teaching aids, tests, teaching methods, study techniques, enrichment, results of research, and other subjects.

The Oklahoma Council of Teachers of Mathematics (OCTM) is one of approximately one hundred and fifty Affiliated Groups of NCTM. The Affiliated Groups and NCTM sponsor professional meetings for mathematics teachers over the state and over the entire United States. It is possible for the members and others to attend meetings several times per year within reasonable driving distances of their schools. OCTM also publishes a Newsletter twice a year for its membership of approximately six hundred.

Membership in such professional organizations makes it possible for mathematics teachers to keep abreast of what is happening in their field. This should be encouraged by the school administration. Institutional subscriptions may be made for schools, libraries, and institutions or wherever the journals are desired for a group. Availability of the journals and other publications of these mathematical professional organizations will be of great value in giving teachers the enthusiasm so much needed in improving mathematics in the schools.

NCTM membership and subscription rates are as follows:

Individual Membership	
Including either journal	$5.00
Including both journals	8.00
Mathematics Student Journal	.50
Student Membership	
Including either journal	2.50
Including both journals	4.00
Institutional Subscription	
Mathematics Teacher	7.00
Arithmetic Teacher	7.00
Mathematics Student Journal	.40
(for 5 or more to a single address)	

OCTM membership rate is $2.00 per year which includes the *Newsletter.*

Address of NCTM
1201 Sixteenth Street, N. W.
Washington, D.C. 20036

Suggested Guidelines for the Improvement of a School Mathematics Program

These Guidelines form a summary of the points discussed in the preceding pages. It is hoped that these suggestions will help the schools of Oklahoma in offering a more adequate program in mathematics for the youth of the state. The Guidelines are not meant to be all inclusive and it is quite probable that schools will want to consider other procedures not mentioned here.

I. The administration and the teachers must assess the resources of the school and of the community before it can decide on the type and scope of the school program which can be offered and supported.

II. The needs of the pupils and of the community in the area of mathematics must also be assessed in order to decide on possible improvements.

III. Give a hard look at the potential of both the administrative staff and of those who will teach mathematics.

IV. If there are inadequacies among those who will teach mathematics, and among those on the administrative staff involved with them, and there will be some in almost any school, make some adequate provision for improving their competence. This is an administrative responsibility and the possibility of releasing the staff for one hour one day a week for in-service work should be considered.

V. Decide on the objectives of the school in terms of mathematics. That is, the content and the characteristics of the content for various school levels, with specific courses to be offered to high school students. Decide whether it will be possible to teach mathematics at two or more levels or whether the school will have to teach a single track to all.

VI. Select the most suitable textbooks in connection with the items mentioned above.

VII. Select and purchase other instructional materials, both for the library and for the individual students. This will depend on the level of classes, types of students and the like, as well as, the amount of money available for this purpose.

VIII. Scrutinize with great care any sort of teaching material designed for mass teaching, particularly as to behavioral objectives and evidence that use of the material for the particular classes or groups of classes will indeed produce the desired learning on the part of the students.

IX. Encourage teachers to join professional organizations, read the journals and other publications, attend the meetings and to become active in the organization.

Recommendation

It is recommended that all schools consider carefully the concept-understanding approach to teaching mathematics at all levels of instruction. It is recognized that the final decision must be made after a careful consideration of the objectives and needs of the school and the community and of the resources of the school and its staff.

Bibliography

I. *Books*

1. American Association of School Administrators, Association of Supervision and Curriculum Development, National Association of Secondary School Principals and the National Council of Teachers of Mathematics. *Administrative Responsibilities for Improving Mathematics Programs,* Washington, D.C.: NCTM 1965.

2. Association for Supervision and Curriculum Development. *Commission on Current Curriculum Developments.* Washington D.C.: The Association, 1965, pp. 46-56.

3. Brown, Kenneth E. and Snader, Daniel W. *Inservice Education of High School Mathematics Teachers.* Washington, D.C.: U. S. Department of Health Education and Welfare, Office of Education, Government Printing Office, 1961, 109 p.

4. Deans, Edwina. *Elementary School Mathematics: New Directions.* Washington, D.C.: U. S. Department of Health, Education and Welfare, Office of Education, G. P. O. 1963, 116 pp.

5. Educational Services Incorporated. *Goals for School Mathematics: The Report of the Cambridge Conference on School Mathematics.* Boston, Mass.: Houghton Mifflin Co., 1963, ix + 102 pp.

6. Miles, Matthew B., editor. *Innovation in Education.* New York: Bureau of Publications, Teachers College, Columbia University, 1964, 689 pp., See Chapter 14, page 329 "School Superintendents and the Adoption of Modern Math: A Social Structure Profile."

7. National Council of Teachers of Mathematics. *An Analysis of New Mathematics Programs.* Washington, D.C.: The Council, 1963.

8. National Council of Teachers of Mathematics. *The Growth of Mathematical Ideas, Grades K-12,* Twenty-Fourth Yearbook. Washington, D.C.: The Council, 1959.

9. National Council of Teachers of Mathematics. *The Revolution in School Mathematics.* Washington, D.C.: The Council, 1961.

10. National Council of Teachers of Mathematics. *The Supervisor of Mathematics; His Role in the Improvement of Mathematics Instruction.* Washington, D.C.: The Council 1959, 10 pp.

11. National Council of Teachers of Mathematics. *Topics in Mathematics for Elementary School Teachers,* Twenty-Ninth Yearbook. Washington, D.C.: The Council, 1964, 384 pp.

12. National Education Association. *The New Mathematics.* Washington, D.C.: The Association, 1964, 10 pp.

13. Schult, Veryl, and Abell, Theodore L. *Inservice Mathematics Education: Promising Practices for Elementary and Secondary School Teachers.* Washington, D.C.: U. S. Department of Health Education and Welfare, Office of Education, Government Printing Office, 1964, 129 pp.

14. National Council of Teachers of Mathematics. *Inservice Education in Elementary School Mathematics.* Washington, D.C.: The Council, 1967, 64 pp.

15. The National Council of Teachers of Mathematics, *Research in Mathematics Education.* Washington, D.C.: The Council, 1967, 132 pp.

II. *Films*

16. National Council of Teachers of Mathematics. *Films in Mathematics for Elementary School Teachers.* 10 films, 30 min., 16 mm. sound, color, Order from United World Films, Inc., 221 Park Avenue South, New York: 10003.

17. National Council of Teachers of Mathematics. Booklet: *Mathematics for Elementary School Teachers.* (To accompany the films listed in 20), The Council, 1965.

18. National Council of Teachers of Mathematics. *Mathematics for Tomorrow.* One film, 29 min., 16 mm., sound, color, (With the Mathematical Association of America), 1965, Order from Audiovisual Sound Studio, 1201 Sixteenth Street, N. W., Washington, D.C.: 20036.

19. School Mathematics Study Group. *SMSG Teacher Training Films.* 30 films, 30 min., 16 mm., sound, color, 1965, Order from Modern Learning Aids, 3 East 54th Street, New York: 10022.

III. *Magazine Articles*

20. Association for Supervision and Curriculum Development. "Mathematics in the Schools." *Educational Leadership,* Vol. 19, (March 1962), pp. 353-389.

21. Kay, Richard and Woodby, Lauren G. "Project Idaho." *The Mathematics Teacher,* Vol. 58, (March 1965), pp. 241-243.

22. National Education Association, Department of Audio-visual Instruction. "The New Mathematics." *Audio-visual Instruction,* Vol. 7, (March 1962), pp. 136-186.

23. Zant, James H. "Using New Media in the Mathematics Classroom." *Educational Technology,* Vol. VI, No. 13, (June 30, 1966), pp. 1-8.

SECTION IV

Mathematics for the Underachiever

Introduction

The problem of the underachiever in mathematics has always been recognized by administrators and teachers in the schools. There has been little success, however, in finding a solution. Administrators should assume the leadership in cooperative planning with teachers. These teachers can contribute to success through the realization that work with these students is an important and worthwhile task.

The challenge of doing an effective job for this particular problem is probably greater than in most other places in the school program. Students must be made to see that both the skills and understanding of mathematics are worthwhile and for them a very practical activity on the school level and beyond. Teachers who can accomplish this with these students will perhaps be the most skillful and professional ones in the schools and their status and rewards should reflect these abilities.

A strange thing about the teaching profession is that prestige and monetary reward go to those assigned to the easiest job, that of teaching the well qualified students. Most of these can learn without a teacher. The challenging job, that of teaching "the slow learners, the lazy, the inept" in too many cases is assigned to the teacher lowest within the hierarchy. This is usually the beginning teacher. What would one think of a doctor who would say, "Isn't it wonderful, I get nothing but healthy patients?" Where in this teaching profession are the members who would rub their hands with glee like a surgeon does when he gets a challenging case? Unquestionably at the school level, that challenging case is the child whom no one else would dare attempt to teach.

We know too little about the psychological and sociological forces which affect these children, but experts are busy working in these areas. We are more aware of some of the explosive aspects of the growth of knowledge in the areas of mathematics and science. This has led us to the realization that today we are living in a world which is typified by Margaret Mead's beautiful words, "Nobody is going to die any more in the world in which he was born."

The Low Achiever

I. *General Characteristics*

There seems to be no general agreement on the criteria by which a student is classified as a low achiever. Some schools seem to begin with a list of students with a low intelligence. In terms of percentiles it is the list of those below the 25 or 30 percentile, on the basis of I. Q. scores with limits of 80 to 90. It is often stated that the group also includes those students who are at least one or two years behind in particular subjects or grades. Until there is more definite agreement, it will be considered that "the low achiever," "the slow learner" and the like will mean approximately the lower one-third of the school population on the basis of intellectual ability and success in the school program.

Decisions on the selection criteria for this group will have to be made by individual schools based on local conditions. The Baltimore City Public Schools make this statement:

"Pupil selection criteria should be clearly and concisely stated. Under no circumstances should this program be considered the dumping ground for students with behavioral problems. Rather, the students selected should be those who, despite being low achievers, have a desire to learn, and even more important, those whose parents want them to learn also.

"The low achiever can best be characterized as students having an intellectual potential at the low-average level, who are generally about one or more years below grade in arithmetic and one or more years retarded in reading. These characteristics should serve as guidelines for admitting students to a basic mathematics program, but not as inflexible criteria, to be rigidly applied." (See pp. 35-36, USOE, *The Low Achiever on Mathematics.*) (Prepared by Lauren G. Woodby).

II. *Motivation*

The attitudes of low achievers toward mathematics, his lack of interest and much of his lack of ability to learn it as it has been presented to him in the past is probably the source of his almost complete lack of motivation for the study of mathematics. It is not surprising that he is uninterested in further efforts on a subject in which he has been unsuccessful for so long.

To insure success by the student in his study of mathematics, the program offered must be changed, new approaches to teaching must be explored and probably more emphasis must be placed on basic understandings and structure of mathematics. Nothing must be neglected which will give the student needed background experience with physical objects and the opportunity to classify and generalize on these observations. It is possible, however, that some of these students should not be asked to generalize until much later in their experience with mathematics.

It does seem to be true, from the limited experiences with the category of students so far, that such things as real life problems, computer oriented mathematics, the use of models in a mathematical laboratory and other audio-visual devices give a much needed motivation to the students. The use of these devices must be explored thoroughly, and we may not know definitely whether these are as useful as anticipated until such careful experimental work has been done.

III. *Content and Organization of Material*

These students' background in both mathematical skills and in the understanding of the basic principles and structure of mathematics will be limited, but of a varying degree. Their ability to learn will also vary widely. For these reasons much of the teaching and learning must be of an individual nature. Unless classes can be made quite small, this will present many problems to the teacher. Advantage must be taken of such individual learning devices as programmed instruction, teaching machines, films (probably short ones), calculating machines and various kinds of models and laboratory equipment. It is suggested that the use of programmed instruction, teaching machines and films be carefully investigated. Industrial training programs have made effective use of these devices for some time in their pre-service and in-service work with employees. They have not only been able to teach these employees the basic knowledge needed on the job, but have also been able to teach them basic concepts of English, science and mathematics. The problems and characteristics of these employees in industrial training programs are not basically different from those present in special programs for the underachievers in the schools.

IV. *Objectives and Goals*

It is important with low achievers, as with all students, that objectives be carefully and specifically stated and that they be behavioral in character. Only in this way is it possible to measure accurately whether or not the students have learned what is intended, that is, whether their behavior has been changed. It is also important to evaluate teaching procedure and teaching materials of any type in terms of ***student learning***. If the children *do not* learn, what has been done in the classroom is at fault; the teaching or the material presented, or both, have been wrong and must be revised until students actually learn what is intended. (See Benjamin S. Bloom, Editor, *Taxonomy of Educational Objectives, The Classification of Educational Goals.)* It is possible, of course, that the objectives set are too high. With the group of low achievers this can happen, because we do not yet know just what should be attempted with this group.

V. *Research and Experimentation*

To meet the educational needs of a very dynamic, fluid and rapidly changing civilization, we must be constantly building into our educational system, at all levels, a substantial effort in research and development so that effective teaching materials, including all media, can extend the efforts of able teachers. At no place in the schools is this effort needed more than in the area of the low achiever. It will require extensive experimentation and design and the close cooperation of sympathetic subject matter specialists.

Recent approaches to the problem of teaching low achievers and research into the psychology of learning indicates that students of low ability can learn more mathematics than they are learning at present. There is also evidence that the students can learn mathematical concepts and processes at ages sixteen to eighteen that they were not able to learn when they were younger. We know that low achievers usually have a short retention span, but in spite of this it has been shown that they *can* learn the sound, practical, everyday mathematics needed for wise management of their personal affairs and efficient performance of jobs.

It is well known that students who are low achievers in mathematics fall into two categories, those who normally learn at a slow rate and those achieving success in mathematics below their apparent potential to learn. Recent developments in Federal aid through the Elementary and Secondary Education Act (ESEA) of 1964, Title I for disadvantaged students and Title III for innovations in curricula development, have made it possible for administrators to study the problem and do some experimental work with teaching materials and methods. It can, therefore, be approached with more care and hope for success.

It will be the purpose of this section to describe some of the experimental work that is already under way over the state and country and to discuss some of the things which a particular school system must consider to get Federal support and to build a successful program for these students. In this section we shall also consider those students who are low achievers in mathematics because they have average ability, but who, for some reason, have not been successful in their work in mathematics. It is true that there are students of high ability, sometimes called "talented students," who should be classified as low achievers because they are not doing as well as their ability indicates they should be doing. This is really another problem which requires different teaching materials, a different method of presentation and a different basic philosophy. What to do with these students who are potential leaders in our society is a basic educational problem which will be discussed in Section V.

A Different Approach Necessary

It is obvious that low achievers have not been able to attain proficiency in the basic skills of mathematics; that is, in addition, multiplication, fractions, decimals and the like, ordinarily expected in the schools. To an equal extent they have been unable to understand the basic concepts of number, measurement and geometry and to apply what little they have learned to everyday problem solving. Yet the standard approach in the schools has been to give more practice in the desired skills, more problems and the like. It is not surprising that these students, who have a long history of a lack of success in this very kind of material, are discouraged and have little interest or motivation in what they are required to attempt again.

Hence we must find a more realistic approach to mathematics for these students and consider carefully the psychological factors involved in giving them the motivation to learn both the skills and the basic principles of mathematics involved in what we hope to teach them. Much of this new approach must be discovered through experimentation with teaching materials and children. But good teachers are already aware of many things that must be done. For example, when students have no knowledge or understanding of borrowing in subtraction, then it is of no advantage to assign him exercises or problems which require this knowledge and understanding. The teacher must go back and teach this section of subject matter in much the same manner as it is taught to younger children. Moreover, she will probably have to teach it to him several times. Problems are also important here and much is being said now about "real life" problems, often collected from business men in the community, but even good students have trouble with word problems. Low achievers are often low in reading ability, a real handicap in solving word problems, and they may have little understanding of the situation being described in the problem (many of them are disadvantaged too). A way of overcoming some of these difficulties is to have these students write some of their own problems. In addition to giving them some badly needed practice in self expression, the words will be simpler and the situation being described will probably be much nearer to the experience of all of them than when the problems are written by adults and handed to the students. This approach has been tried with considerable success by teachers in the schools.

The mathematics laboratory has been used successfully with students who are low achievers. Indeed it is a necessary element when the children are culturally disadvantaged. It is difficult for a college-educated, middle-class teacher to imagine the deprivation a slum child faces from birth. A teacher of mathematics must also consider the low achiever's paucity of experience with the manipulation of the objects of the physical world. The mathematics laboratory should contain things, all sorts of things, to be played with, handled, classified, counted, measured, weighed, etc., so that experience can be gained with the objects and so that basic principles of number and geometry can be illustrated and identified. Under the direction of a skillful teacher many of the concepts can be illustrated and learned. Whole group teaching should probably be held to a minimum in the mathematics laboratory with the teacher's efforts devoted to small groups or individuals.

This description and the illustrations of a new approach to teaching low achievers are not intended to be inclusive or even effective in particular situations. Much depends on the purposes of the school, the personality and preparation of the teacher and many other factors. Selecting the most useful approaches will depend on much experimentation in the schools, research in the psychology of learning; as well as, size of class, size of room, equipment available to the teacher and time for the teacher to do a professional job in a very difficult and often trying situation.

The Mathematics Program for Low Achievers

A recent publication of the U. S. Office of Education, *The Low Achiever in Mathematics,* already referred to, gives a fairly full discussion of the problems of the low achiever with recommendations and guidelines. This will be useful to schools interested in the type of subject matter needed for these students. This publication includes definite suggestions about mathematics programs for the under-achiever. However, the mathematical content of various programs being used is not yet clear and whether or not they will be successful with the students has not been determined. We must, if at all possible, give these students adequate proficiency in the skills of arithmetic (number), of measurement and of informal geometry. These have been referred to earlier as "sound, practical, everyday mathematics;" there is also a need for basic understanding of the principles of number and of geometry so they will be able to solve problems and also to enable students to reconstruct the processes when their memory of rules and procedures become hazy.

Effective procedures and the teaching material which guarantee the teaching of these skills and concepts are not yet available, since little experimentation with developing new programs for these children has been done. There are programs underway which seem to be presenting primarily material involving manipulations and skills to these students. To aid in motivation many "real life problems" are assembled, and when available, calculating machines, computers and mathematical laboratories are used. While these motivating devices seem to be effective in getting students to learn the desired skills, it cannot be assumed without much more evidence than we now have, that these students will generalize these activities into the basic principles and structure of mathematics. Even good students often fail to accomplish this.

One of the modern aims for the students who do well in the schools is to become familiar with and understand the basic principles of number (there are only eleven of them) and those of intuitive geometry including measurement. This is called the structure of mathematics and is the basis for building an understanding that will enable the students to make use of the basic operational skills and the solving of problems.

The mathematical structure for the low achiever in mathematics should probably be as definite and should be presented in a logical manner. It may well be less complete, more intuitive and less sophisticated than that ordinarily used in the schools, but students need something to make their work in mathematics meaningful and understandable. While much more experimentation must be done in this area, there is some evidence that the structure ordinarily used for the better students can also be used successfully for less qualified ones, if more time and illustrations are provided and if the pace is slower.

For example, the Muskogee, Oklahoma Public Schools have obtained a grant under ESEA Title I for a pilot program in Algebra I under which "deprived and disadvantaged students" will attempt an Experimental Program in Algebra I using the SMSG text, *An Introduction to Algebra, Parts I and II* for a two year period. Other instructional materials, including programmed instruction, films, visuals, transparencies and the like will be used. This text book, *An Introduction to Algebra,* covers the same topics as are covered by the SMSG, *First Course in Algebra,* which was written for the college capable group. However, it is designed to take twice as long to do so.

Tentative Suggestions on Content of Mathematical Materials or Goals for Underachievers

At the risk of over simplification and the accusation of being premature, two sets of lists of content objectives will be included here. This is done for purposes of clarification and discussion and full agreement is not sought or expected.

The program in mathematics for underachievers should be designed so that the students will gain competence in:

1. Development and proper use of vocabulary
2. Numerical understanding and reasoning
3. Computation
4. Discriminatory thought processes
5. Reading and interpreting graphs and tables
6. Understanding and use of geometric concepts (including measurements)
7. Giving formal and axiomatic expression to concrete ideas
8. Problem solving, involving, among other things, those encountered in actual life situations
9. Some practical applications in the use of mathematics

The following list is a quotation from the USOE pamphlet listed above, *The Low Achiever in Mathematics* and is included in a Position Paper entitled: "Mathematics for Low Achievers: Responsibilities of School Administrators," pages 33-37, by Dr. George B. Brain, Dean of the College of Education, Washington State University, and formerly Superintendent of the Baltimore City Public Schools.

Objectives of the Basic Mathematics Program

The program developed should be broad in scope, flexible in nature, and adaptable in application to each student's needs. Here are the objectives set forth for courses for low achievers in Baltimore:

1. To increase the student's skill in fundamental operations in arithmetic
2. To develop his ability to meet mathematical situations effectively in the home, school, business and community
3. To enable him to use simple formulas, equations, ratios, and proportions
4. To teach him essential aspects of informal geometry as related to real-life situations
5. To increase his understanding of direct measurement
6. To teach him elementary techniques of problem-solving
7. To help him develop a vocabulary rich enough to understand and express mathematical ideas in daily life
8. To develop his ability to think through a quantitative situation, make sound judgments about it, and appraise the reasonableness of his judgments
9. To help him to become a more intelligent, more critical consumer
10. To develop his appreciation of the role that mathematics plays in making advance in the physical world
11. To prepare him adequately for future courses in mathematics

"In accordance with these objectives, criteria for student admission, selection of staff, training of staff, teaching materials, program evaluation, pupil promotion, remedial work, and class size must be determined under the leadership of the school administrator."

Teachers for Low Achievers

As has been stated, the attitudes for teachers for the underachievers is important to the success of any program. If the teachers have successful experiences in the schools and a basic knowledge of mathematics, then the needed preparation will be primarily in selection of materials and methods of presentation. The teacher will need to determine the background and knowledge of the students in the class. Since local conditions and the classes even in the same school vary so much, teaching materials, especially problems, may have to be written or obtained from the community by the teacher. The use of visual aids and other media must be selected carefully by the teachers to fit the objectives of the course for a particular level and for a particular situation. For continued use of all media material the teacher must experiment, find exactly what the media proposes to teach (it is seldom included with the description of the material), try it out in the classroom, and then test to see whether or not the students learn the objectives.

Most of this is in the form of extra work and checking, which the experienced teacher will know how to do, if sufficient time is available. Hence, classes with this type of student should be small to allow for much individualized instruction and the teacher should have enough time available outside the class to do the necessary things for good instruction. The Federal assistance available for work with low achievers also includes a percentage (ten percent) to be used for teacher preparation and consulting. If teachers have had successful experience in the schools, it seems probable that a short workship of two or three wesks before the classes start plus some kind of an in-service program throughout the year will probably be sufficient to enable them to do a good job. Schools applying for assistance in their programs for underachievers should be sure to ask for the item for training programs for teachers and to provide some consulting service.

Guidelines for Preparing Instructional Material in Mathematics for the Low Achiever

The USOE pamphlet, *The Low Achiever in Mathematics*, pp. 89-90, already referred to, points out that good instructional materials are sorely needed for below-average students. The following guidelines are based on, but not quoted, from the discussion included in the report:

1. A national team of specialists from the disciplines of mathematics, mathematics education (including teachers) and psychologists should suggest appropriate course content.

   In the absence of such suggestions and probably after they have been made, the many adaptations necessary for local use must be made by administrators and teachers of the individual school systems.

2. New approaches should be followed in designing the experimental material.

   It is questioned whether a slowed-down version of mathematics for the college capable student is appropriate for the slow learner.

3. Material should provide for the development of basic mathematical understandings, especially those essential for vocational competence.

   "Understandings essential for vocational competence" are difficult to be certain about in our rapidly changing world. A worker should possess mathematical skills and understandings adequate to qualify for admission to the retraining programs for new or different jobs.

4. Opportunity for success should be a major aim in the design of learning materials.

5. The learning material should be graded in content.

6. Study units for the low achievers should be short.

   The student needs the sense of accomplishment derived from completing a task and will profit by the change of pace provided by short units.

7. Special units for work-study programs should be developed.

8. Materials should provide a varied approach to the development of mathematical concepts and skills.

9. When possible, or if possible, experimental materials should be developed by expert writers.

10. Evaluation of the experimental materials should be conducted in a variety of schools and classrooms.

Experimental Programs in Mathematics for Low Achievers

Descriptions of experimental programs now being developed over the country will be useful to schools interested in initiating such programs. None of these are yet fully developed and not much of the actual teaching material is available for inspection. We have some knowledge of a few of these new programs, but many of them have not been publicized.

There are two programs for low achievers established in Oklahoma, one in the Oklahoma City Public Schools called High School General Mathematics. This program completed its first year in June 1967. A second one was developed in the Muskogee Public Schools and class work started in September 1967. Brief descriptions of these two programs and some others over the country will be included in an appendix appearing at the end of this section. These programs follow many of the suggestions included in this section and they should be useful and helpful to many teachers and administrators. There is a national organization called the Concepts and Applications Mathematics Project (CAMP) which expects to hold writing conferences over the country and to establish a central organization to exchange information on what is being done. This group has not been completely funded yet, but this is expected. When the organization is complete the central offices will be at Central College, Pella, Iowa. Schools may want to write them at this address.

Federal Support Available

It has been mentioned already that support may be obtained by application to Titles I and III of Public Law 89-10, ESEA by individual schools. These proposals must be written carefully and the proposed programs to be offered should be described in detail. A realistic budget should be included sufficient to carry out the proposed program. Administrators and teachers are encouraged to investigate this possibility carefully. The State Supervisor of Mathematics is a valuable source of information.

Guidelines for Establishing a Mathematics Program for Underachievers

For schools considering the possibility of organizing a special program in Mathematics for Underachievers, the following statements are suggested as guidelines for teachers and administrators.

1. Analyze the school and the community for the need of special programs for underachievers.
2. Examine programs in operation in the state and nation and make a decision on the mathematics needed, the methods available, including new media, and what experimentation can be made available.
3. Set a policy for determining the students who will be assigned to the course on the basis of achievement, ability, social background, teachers' judgments and the like.
4. Assign the teachers, but consider the fact that this sort of teaching is probably the hardest to do successfully in the school. Hence, if possible, it should carry *prestige* and *rewards.*

5. Consider and improve, if possible, the attitudes of both teachers and students.
6. Consider motivating factors which can be integrated into the curriculum and classroom instruction. Some of these are modern educational technology including the use of calculating machines, new educational media, a mathematics laboratory, classroom activities and experiences with objects in the physical world, use of community resources, and the like.
7. Consider the possibility of Federal support for the program to be developed. Investigate this carefully and include in the proposal a request for funds to be used to prepare teachers for this new kind of responsibility.
8. Make a special effort to give the teachers time and provide help for them through workshops and in-service and consulting service to enable them to do an effective job. The use of Teacher Aides, when possible, and Team Teaching will help teachers in answering many questions asked by this type of student.
9. The school administrator must provide opportunities for teachers to engage in research and experimentation. Do not make final decisions on the type of program needed and the method of presenting it to the students without a considerable amount of experimentation on the local level.

Though the problem of the underachiever in mathematics has always been with us, little has been done in developing teaching programs and materials for this special group. The emphasis for the past ten years has been on mathematics for the college capable student. While there is some evidence that much of this material can be useful in working with the underachiever, there is still the problem of rewriting the material at their level of ability, reading skills, the use of actual situations, the use of more teaching aids and many other things. Furthermore, motivation must be considered and student and teacher response to the newly developed materials may well be the deciding factor in what is kept and what is discarded in these programs. Experimentation with materials, with methods of presentation and the like may well be more useful and necessary in working with the underachiever than with other types of students.

Bibliography

I. *Books*

1. Educational Testing Service. *Survey of Mathematical Education, The Causes of Student Dropout and Incompetence at Elementary and Secondary Level.* Princeton, N. J.: E. T. S., Oct. 1955, (Carnegie Fund).
2. Esso Standard Oil Co., Education Division. *Yes - No - One - Zero.* New York: Esso Standard Oil Co., Free.
3. SMSG. *Introduction to Algebra, Parts I and II.* Pasadena, Calif.: 91105, A. C. Vroman, Inc., 367 South Pasadena Avenue, 1961.
4. SMSG. *Introduction to Secondary School Mathematics, Volumes I & II.* Pasadena, Calif.: 91105, A. C. Vroman, Inc., 367 South Pasadena Avenue, 1961.

5. SMSG. *Mathematics Education for Below Average Achievers.* Pasadena, Calif.: 91105, A. C. Vroman, Inc., 367 South Pasadena Avenue.
6. U. S. Office of Education. *The Low Achiever in Mathematics.* Prepared by Lauren G. Woodby, Mathematics Specialist, U. S. Department of Health, Education, and Welfare, Office of Education, OE 29061, Bulletin 1965, No. 31, 96 pages. (For sale by the Supt. of Documents, U. S. Government Printing Office, Washington, D.C. 20402.)

II. *Magazine Articles*

1. Braunfeld, Peter and Martin Wolfe. "Fractions for Low Achievers," *The Arithmetic Teacher**, Vol 13, No. 8, (Dec. 1966) pp. 647-655.
2. Callahan, John J. and Ruth S. Jacobson, "An Experiment With Retarded Children and Cuisenaire Rods," *The Arithmetic Teacher.* Vol. 14, No. 1, (Jan. 1967) pp. 10-13.
3. Cleveland, Gerald A. and Dorothy L. Bosworth. "A Study of Certain Psychological and Sociological Characteristics as Related to Arithmetic Achievement," *The Arithmetic Teacher,* Vol. 14, No. 5, (May 1967) pp. 383-387.
4. Elder, Florence. "Mathematics for the Below Average Achiever in High School," *The Mathematics Teacher**, Vol. LX, No. 3, (March 1967) pp. 235-240.
5. Fremont, Herbert and Neal Ehrenberg. "The Hidden Potential of Low Achievers," *The Mathematics Teacher,* Vol. LIX, (Oct. 1966) pp. 551-557.
6. Hammett, Helen. "Evaluating and Reteaching Slow Learners," *The Arithmetic Teacher,* Vol. 14, No. 1, (Jan. 1967) pp. 40-41.
7. Johnson, Donovan A. "Next Steps in School Mathematics," *The Arithmetic Teacher,* Vol. 14, No. 3, (Mar. 1967) pp. 185-189.
8. Newport, John F. "Are Almost All Children Disadvantaged in Mathematics?" *The Arithmetic Teacher,* Vol. 14, No. 1, (Jan. 1967) pp. 21-23.
9. Paschal, Billy J. "Geometry for the Disadvantaged," *The Arithmetic Teacher,* Vol. 14, No. 1, (Jan. 1967) pp. 4-6.
10. Sister Gilmary. "Transfer Effects on Reading Remediation to Arithmetic Computation When Intelligence is Controlled and All Other School Factors Are Eliminated," *The Arithmetic Teacher,* Vol. 14, No. 1, (Jan. 1967) pp. 17-20.
11. Sweet, Raymond. "Organizing a Mathematics Laboratory," *The Mathematics Teacher,* Vol. LX, No. 2, (Feb. 1967) pp. 117-120.

There are many articles in many periodicals about the teaching of low achievers. Many are general in character and those which deal with mathematics for low achievers, if they deal with specifics at all, are limited to discussion of specific topics with perhaps something about methods, equipment and content. This is useful, but it may be considered as a first step only.

---

*The Arithmetic Teacher and The Mathematics Teacher are published by the National Council of Teachers of Mathematics, 1201 Sixteenth St. N. W., Washington, D.C. 20036.

It is suggested that teachers and administrators keep on the lookout for new articles and for those which have the potential of helping in the classroom. Many more articles will appear over the next few years. It is a very popular field and soon articles will appear which will describe actual teaching experiences and actual programs for these students.

APPENDIX

Short Descriptions of Programs in Mathematics for Low Achievers

HIGH SCHOOL GENERAL MATHEMATICS

William J. Coffia

Supervisor of Mathematics

Oklahoma City Public Schools

Goals and Objectives

A high percentage of the students enrolled in high school general mathematics enter the labor market upon graduation or drop-out. Therefore, it is highly important that they have the mathematical competence that is necessary for gainful employment. Thus, the major purpose of this program is to provide opportunities that will enable students to continue to develop some degree of mathematical competence that will help make them self-supporting and contributing citizens.

Unique Procedures

The uniqueness of the Oklahoma City program lies in the collection of "real life" problems peculiar to businesses in Oklahoma City. A considerable number of business concerns in Oklahoma City provided typical problems that their employees encountered on the job. In most cases the solution required the use of mathematics to some extent. The problems were printed on company stationery and incorporated into the general mathematics course, thus hopefully, illustrating to students the relationship between school experiences and potential vocational experiences.

Another important feature is the use of computational devices such as a slide rule and in some cases a print-out type desk calculator. The idea is to provide a tool for computation thus relieving the student of some of the drudgery involved in computational procedures.

The procedure a student follows in dealing with a problem is to actually show the solution through drawing a flow chart that indicates all of the necessary steps, including arithmetic operations. After the flow chart is prepared the student then goes to a computational device for doing the necessary arithmetic.

Achievement of Goals

For some time teachers have used the "real life" problem approach to teach general mathematics. The students were, with a few exceptions, juniors or seniors in high school. During the month of September the students were given form 2A of the Sequential Tests for Educational Progress. Near the close of the school year the same test was administered and the results compared with the pre-test scores. Also, the pilot classes were compared with other classes. In addition to the above, the teachers in the pilot classes made an estimation of student attitude as compared with students in classes of previous years.

These questions were formulated by the Oklahoma City Public School Project in High School General Mathematics. Some of these can be answered, but all of them must be answered to achieve an effective program in mathematics for low achievers.

1. Name some characteristics of the slow learner.
2. What are some interest areas of the students who are *slow-low* achievers in mathematics?
3. How can these interest areas be incorporated in our classes as a teaching tool?
4. Name some personal characteristics teachers of "slow learners" should have.
5. Should a teacher be identified as a teacher of slow learners?
6. What should the professional training of teachers of slow learners include?
7. How can we most effectively transfer to the student an enthusiasm for mathematics?
8. How do we as teachers fight lack of interest in our students, particularly the slow learner?
9. In these low achiever groups how can the teacher raise their "success" level?
10. How can teachers be inspired to want to participate in slow learner problems and progress?
11. What might be a reasonable approach to make administrators aware of this pressing problem?
12. Suggest some methods for parent training and participation in this program.
13. Business (long a loud critic of the mathematics background of the labor market) could and should be an effective contributor to the motivation of the slow learner. How can the schools become a partnership in this project with business?
14. In our effort to keep youngsters in school how can we help them see beyond the immediate desire, see the skills needed to make a living in our complex world?
15. To prevent life long public charges how can we help students to acquire abilities that make them employable in skills not duplicated by machines?

A RESEARCH PROJECT IN MATHEMATICS #1

W. L. Waid

Mathematics Coordinator

Muskogee, Oklahoma Public Schools

Title:

A two year course in Algebra I for the development and improvement of high school mathematics skills through an individualized program and audio-visual format for low achievers of average or slightly below average ability. (Grades 9 and 10).

Objectives:

To increase the student's skills in fundamental operations of mathematics

To develop ability to meet mathematical situations effectively in the home, school, business and community

To prepare him adequately for further courses in mathematics and thereby reduce the number of drop-outs from mathematics

To evaluate a two year Algebra I program against the traditional one year program now scheduled for low achievers

Identification:

Stanford Achievement (Advance Modern Mathematics Concepts)
Iowa Algebra Aptitude Tests (Revised)
SAT (These have been
Otis Mental Maturity given in the past
California Mental Maturity two years)

Description:

A full two year course in Algebra I for students whose achievement has been handicapped by inadequate mathematical background. Essential mathematical skills are to be developed and applied, with provisions for remedial instructions when needed. The classes will progress at a speed paced by the students. Continuous teacher and student evaluation is an aspect of the project.

A syllabus will be planned and developed by principals, heads of departments, teachers, counselors, and the mathematics coordinator.

Consultants:

A university consultant will conduct in-service sessions.

Materials:

Text

Introduction to Algebra (S.M.S.G.), Book I and II, A. C. Vroman, Inc., (written for low achievers)

Visual aids and supplements
- Programmed Practices for Algebra
- Films
- Visuals for Modern Algebra
- Duplicating masters
- Viewer Stage for visuals

Overhead transparencies and visual stage to illustrate difficult concepts, improve motivation, enrich motivation, and aid in reviews.

Evaluation:

Progress tests for Algebra
Lankton 1st year Algebra Test (revised)
Student and Teacher
Parent reaction

A RESEARCH PROJECT IN MATHEMATICS #2

W. L. Waid

Mathematics Coordinator

Muskogee, Oklahoma Public Schools

Title:

A three year General Mathematics course to help students with low abilities learn practical as well as modern concepts and skills. (Grades seven, eight, and nine)

Objectives:

To equip low ability students with basic mathematical concepts and skills needed in vocational and everyday affairs.

To develop their appreciation of the role that mathematics plays in making advances in the modern world.

To prepare them adequately for further courses in mathematics.

To evaluate new material and teaching techniques; against material now in use.

Identifications:

Stanford Achievement Tests (Intermediate Modern Mathematics Concepts)

SAT
Otis Mental Maturity
California Mental Maturity

(These have been given in the past two years)

Description:

A research project carried out in the seventh grade this year and followed up in the eighth and ninth grades with a series of materials written for low ability students. There will be one class using the experimental materials and a control group using the adopted text now in use. The instruction will be inductive in nature with the emphasis in the use of the environmental materials of many kinds and relying heavily upon the discovery approach. A variety of techniques will be used to help students gain new confidence in their ability to learn. No homework will be required.

Consultant:

A university consultant will conduct in-service sessions.

Materials:

- Text
- *First, Second,* and *Third Courses in Basic Modern Mathematics,* Eichols, O'Daffer, Brumfiel, Shanks, published by Addison-Wesley Publishing Co., Inc., (written for the slow learner)
- Visual aids and supplements
- Transparencies
- Film Strips
- Films
- Teacher's Guide
- Programmed materials

Evaluation:

- Progress Tests for Basic Modern Mathematics
- SAT—Advanced Modern Mathematics Concepts (Given Sept., Feb., and May)
- Continuous teacher and student evaluations
- Parent evaluation—at end of first semester and at the end of the year

THE LOW ACHIEVER — CAN HIS VIEW OF MATHEMATICS BE CHANGED?

John Bradford, Coordinator of Mathematics, K-12

Terry Shoemaker, Mathematics Instructor

Jefferson County Public Schools

158 Yarrow Street

Lakewood, Colorado

In the ferment of change in mathematics education the low achieving student—the potential dropout passing time in courses unsuited to his needs or interests—has been essentially bypassed. New courses in mathematics for college-bound students have been widely acclaimed but very little effort has been made to provide better courses for low achievers. Businessmen complain bitterly that new employees are not prepared for basic practical problems that arise daily. Thousands of students graduate from high schools annually with deficient arithmetic skills, many below that of the average eighth grader. Most of these low achieving students have a fear of mathematics and a distaste for it nurtured by years of frustration due to lack of understanding and interest. Most can see little use for mathematics, can make no sense out of it, and so find it frustrating and deadly dull. *This is a logical and intelligent view based upon years of experience.* Can it be changed?

During the school year 1966-67, the Jefferson County, Colorado School District began an effort to devise a senior high school mathematics course which would change the view of the low achieving student and develop interest, understanding, and practical mathematical skill. With limited financial support from Title I, Public Law 89-10, ESEA, Mr. Terry Shoemaker, a teacher at Alameda High School, began the task of building the course in August, 1966.

Mr. Shoemaker began by considering characteristics and attitudes of the students who would be enrolled in the course. A total of seventy-seven juniors and seniors with an average high school level California Achievement Score of 8.4 were enrolled. Generally these students have experienced frustration and failure in mathematics and have developed negative attitudes toward it. Many have difficulty with reading and comprehension. Many tend to jump to conclusions on the basis of little evidence. Interest span is usually limited to ten to fifteen minutes at any one activity. Absence rates are high. These students have a pattern of living and learning based on their view of the immediate future. Unlike the college-bound student, they desire immediate results and evaluation of their work. They tend to regard themselves as "failures" and "misfits" in school and nearly all have problems which the school does little to meet. Some express these problems by antisocial behavior which leads the school to classify them as discipline problems.

With these characteristics in mind, Mr. Shoemaker began the task of constructing a program which could succeed in this atmosphere and help to change these students' view of mathematics and themselves. Major characteristics of the program are outlined in the remainder of this article.

The "Unit Per Day" Plan

In order to cope with frequent absences, the students' need for immediate evaluation, and a limited span of interest, the "unit per day" plan was devised. Under this plan each day's work is treated as a separate entity which can be completed during the class period. There is no frustrating carryover of assignments. A student may be absent one day and pick up with a new activity the next. Each day's work is planned with at least two changes of type of activity to keep up interest and attention. No textbook is used. Mr. Shoemaker has designed over 400 one page worksheets which are given to the students as they are needed. These present a wide variety of activities in unusual formats from local business problems to crossnumber puzzles. Reading is kept to a minimum and often instructions and problems are given in short phrases of four or five words. Students are asked to evaluate each work sheet on the basis of difficulty and interest through the use of a rubber stamp form. Naturally this involves them in making decisions about the materials to be used in their course.

Electric Printing Calculators

An electric printing calculator has been provided for each two students. The calculator is used as a device to illustrate basic arithmetical calculation, to assist the student in checking his own work, and to help him discover his own error patterns. Such ideas as decimal point location, estimation, and introduction of machine programming are easily conveyed via the calculator. ***Learning to master the operation of the machine gives the students a feeling of pride in their achievement.***

Local Business Problems

Mr. Shoemaker began to contact local businessmen in August before the start of school. He briefly outlined the purpose of the course he was constructing and then solicited their help by asking for the most common mathematical problems which employees faced in their business. These problems were submitted on the businessman's stationery or forms and reproduced for classroom use. Problems from businesses in the local area, whose names students recognized emphasized the practical nature of mathematics. Those businessmen contacted were, without exception, interested, cooperative, and anxious to help. A local supermarket manager volunteered to give students a tour of the store explaining its operation and he administered the weekly checkstand operator's test to the visiting class. Other businessmen have visited the school to talk with classes about mathematical competencies needed for employment. Still others are serving on an advisory committee to help evaluate the course.

Flow Charting

The process of realizing and planning the sequence of steps in problem solving is an important skill for students to acquire. The flow charting technique used to program electrical digital computers is a powerful device to illustrate logical, sequential thinking in problem solving. Students begin flow charting with non-mathematical situations such as "getting ready for school in the morning" or "calling a girl for a date" or "starting a car" or some other familiar sequential operation of interest to the student. After students realize that problems can be flow charted and gain skill at constructing the linear diagrams, problem solving appears in a different setting than ever before. Students proceed to flow chart various mathematical operations such as finding the least common multiple of a set of numbers or addition of fractions. A pupil having difficulty in forming an abstract generalization in his mind can see the complete sequence pictorially in a flow chart. Flow charts are used effectively to introduce an idea, for review and for reference.

Mathematical Experiments

Individual and small group experiments with inexpensive physical apparatus have been designed to provide students with the opportunity to explore important concepts and ideas by using their own ingenuity, developing their own means of comparison, developing their own means of recording and analyzing data, producing their own formulas and reaching conclusions based upon physical evidence. One experiment involves developing a means of measuring and comparing such diverse elements as a large turkey feather, a rock, a glass, some ceramic tiles, a quantity of sand, a paper card, a nail, and a quantity of powdered soap. Students must construct their own apparatus to enable them to make comparative statements about the objects to be measured. Such basic ideas as finding the ratio of the circumference to the diameter of a circle are treated experimentally and related to such real problems as why a man's speedometer may be incorrect when new snow tires are used on his car.

Multi-Sensory Aids

A wide variety of auditory, visual, and tactile experiences have been used to provide interest and variety. A tape recorder with earphones available or dictating equipment may be used for activities such as practice and drill on basic addition and multiplication facts, additional help for a poor reader, instructions for a previous day's assignment, instructions for student projects such as constructing a set of Napier's bones, and enrichment work for more able students. Recorded commentaries may be made for filmstrips, experiments, overhead projections, games, puzzles, etc. The same verbal presentation that a teacher might plan to give in class may be made more interesting to students by tape recording it. With the assistance of a tape recorder more time is available to the teacher to assist individual students.

The overhead projector provides the possibility of superposition of diagrams, effective use of color, and pre-preparation of charts, tables, graphs, etc., which may be used countless times. Students enjoy assisting in preparation of projectuals on a sheet of clear acetate. Discussion of particular portions of a student work sheet can be greatly enhanced by a projectual of the work sheet for all to view.

Slide rules, geoboards, Cuisenaire rods, puzzles, curve stitching, geometric tiles, calculators, scales, balances, etc., provide additional visual and tactile experiences which appeal to students and greatly enhance learning.

Although much evaluation remains to be done, the result of this course as measured by subjective observation of student interest and reaction and teacher evaluation of student growth in mathematical skill and understanding, gives substantial evidence of success. Teacher training is already under way and the Jefferson County Schools plan to extend the program to other high schools in the system. The program is being further developed and revised. The work already completed holds a strong promise that a major breakthrough may be made in providing a valuable course for low achievers.

IMPROVEMENT PROJECT 3—HANOI TOWER

Jefferson County Public Schools

1580 Yarrow Street

Lakewood, Colorado

HOW USED: The project has been used for enrichment, available only to students who have completed their daily work. After a few days (because of the high interest), all students were given an opportunity to work on it. It could be used with high achievers or advanced mathematics classes with the additional problem of determining the algebraic solution.

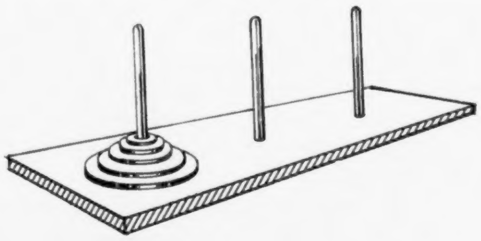
MOTIVATION: Very high, in fact, much higher than any other project used thus far this year. I recommend it for any student in grades five through twelve.

LESSON INCLUSION: The most important lesson is problem solving and pattern recognition. For the high achiever: the task of finding an appropriate rule for the least number of moves to transfer the circle from one peg to another be included.

CONSTRUCTIONAL MATERIALS: A board with three pegs. Several washers or hardboard discs of different sizes.

STUDENT DIRECTIONS: STUDENT DIRECTION CARD

> *OBJECTIVE:* Transfer all discs from one peg to any other peg by moving one at a time.
>
> *RULES:* Move one disc at a time. Never place a larger disc on top of a smaller disc. Find the least number of moves necessary for transferring all the discs from one peg to another.



Jefferson County Public Schools
Lakewood, Colorado

SENIOR HIGH MATHEMATICS

RENAME THE FIRST NUMBER

I. 24 = 4 × ____ = ____ × 2 = ____ × 1 = 3 × ____

II. 30 = 6 × ____ = 2 × ____ = 10 × ____ × 3 = 2 × 3 × ____

III. 72 = ____ × 9 = 6 × ____ = 2 × 9 × ____ = 3 × ____

IV. 15 = 3 × ____ = 5 × ____ = ____ × 45 = 3 × ____ × 5 = 9× ____

V. 64 = 8 × ____ = 4 × ____ × 4 = 2 × ____ = 2 × 2 × 2 × 2 × 2 × ____

VI. 1 = 15 + ____ = 12 + ____ = 40 + ____ = ____ + 7

VII. 10 = ____ + 3 = ____ + 10 = 70 + ____ = 140 + ____

VIII. 7 = 21 + ____ 70 + ____ = 7 + ____ = ____ + 5

IX. 12 = ____ + 2 = ____ + 5 = ____ + 7 = ____ + 10

X. 15 = 15 + ____ = 135 + ____ = 105 + ____

* * * * *

Using counting numbers and zero, complete the information with the given limitations.

12 − ____ = ____ + ____

Number of Ways

(a) Using any counting numbers and zero .... ____

(b) Using only even numbers .... ____

(c) Using only odd numbers .... ____

(d) Using only multiples of 3 (including zero) ....____________

(e) Using multiples of 7 (including zero) ....____________

(f) Using multiples of 6 (including zero) ....____________

WYNNE HIGH SCHOOL

Wynne, Arkansas

CALCULATORS IN GENERAL MATHEMATICS

The School Board of Wynne Public Schools authorized Gene Catterton to write and submit a proposal to USOE for a planning grant under Title III, ESEA, during the summer of 1966. The proposal was accepted and Wynne Schools received a grant for \$23,000.00. This made it possible for eight teachers to work together six weeks during the summer planning for the experimental project. One of the major accomplishments of this planning was the development of approximately five hundred pages of material to be used with low achievers in ninth grade general mathematics. Another major accomplishment was the developing of plans for submitting a proposal for an operational grant to the USOE.

Mr. Catterton prepared the proposal for the operational grant and it was approved by the USOE for \$80,396.00. This is being used to support the experimental project in five public schools of Arkansas. The participating schools are:

Dial Junior High School	Pine Bluff, Arkansas
Northside High School	Fort Smith, Arkansas
Wynne High School	Wynne, Arkansas
Lewisville Public School	Lewisville, Arkansas
Annie Camp Junior High	Jonesboro, Arkansas

In each of these schools there are two to five classes being taught using the material developed under the planning grant and using calculators and other equipment purchased with funds of the operational grant. There are three hundred and seventy-two students involved in fourteen classes with five different teachers.

The students in these classes have been pre-tested and will be post-tested to aid in evaluation of the project. Also, teachers of nearby schools have been extended an invitation to visit one of these schools and observe the program in operation, and then to evaluate what they see.

The teachers involved in this project meet together once each month and discuss the progress being made and to exchange ideas. These teachers agree unanimously that the motivation is extremely high in these classes and that an entirely different attitude prevails in these classes than had been present in previous years. This project extended through January 1968.

MATHEMATICS AND THE SLOW LEARNER

Idaho Falls, Idaho School District No. 91, has not been fortunate enough to have available sufficient funds to secure desk calculators and other special type programs to be used with the slow learner so we have had to resort to other ways of attempting to stimulate the interest of this type of student in mathematics.

The program, which we feel has been quite successful this last year, has consisted of the presentation of a variety of topics. We have used, as a basic guide, *Foundations of Mathematics,* by Wiebe, published by Holt, Rinehart and Winston. This was supplemented very frequently with topics such as: "Graphing Linear Equations," "Determining the Equations from Graphs," "What's My Rule" games, "Problems Most Easily Solved Through the Use of Venn Diagrams", "Working with Exponents," and the use of logarithms and the slide rule.

Quite a bit of work was done with magic squares. Informal geometry was presented and the use of peg boards and Geo-boards was stressed. We used many of the materials developed by the Madison Project, and especially the models developed by working with negative numbers and the multiplication of fractions.

It was felt that this program can be improved upon, but it would go far toward developing the understanding and interest of the low achiever in mathematics.

WALLACE S. MANNING
Math Coordinator
School District #91
150 North Water Avenue
Idaho Falls, Idaho

Robert A. Hansen, Director
Office of Information, Planning, and Research
Fresno City Unified School District
Fresno, California

The following is a rough listing of the techniques presently being used with the low achievers in the Fresno City Unified School District.

PILOT PRACTICES WITH THE LOW ACHIEVER IN MATHEMATICS

1. Use of preliminary edition of *Probability for Primary Grades,* an SMSG text. Being used with good success in grades 1, 2, and 3.
2. Use of *Probability for Intermediate Grades,* preliminary edition of SMSG text. Just starting its use.
3. Used the text entitled *Mathematics and Living Things* in several eighth grade classes. This text uses biological experiences to introduce and enhance the learning of mathematical concepts.
4. Use of three SMSG texts, *Mathematics Through Science,* which use physical science experiments to use mathematic concepts.
5. Saturated two ninth grade math classes with calculators, one per pupil. Some interesting attitude changes.
6. Change ninth grade general math to "free wheeling" lab experiences. Included in the room: calculators, many levels of program texts, abaci, etc.
7. Change tenth grade general math to a completely programmed situation giving individuals in class a variety of seven choices of texts.
8. Encourage interested low achievers to use a student operated computer. (CDC G-15)

9. Senior arithmetic project presently in the fifth year. Using a career approach, our most successful venture.

LONG BEACH UNIFIED SCHOOL DISTRICT PERSONALIZED MATHEMATICS INSTRUCTION

Senate Bill 28, Article 5, State of California

A Model Demonstration Program for Under-Achieving
Eighth Grade Pupils

Preliminary Report
to
Vernon A. Hinze, Assistant Superintendent of High Schools

Approximately three hundred eighty-four potentially capable eighth grade boys and girls in the Long Beach Unified School District are presently participating in a model demonstration program for personalizing mathematics to meet the needs of the under-achiever.* The program was launched the first day of the spring semester (1967).

The innovative and exemplary features of the program center around (1) the personalizing aspects of the teaching team (comprised of the coordinating teacher, the classroom teacher, and six teacher aides from California State College at Long Beach), (2) the laboratory atmosphere created by supplying each student with his personal tote tray of materials and equipment, and (3) by the motivating environment to be provided by a mobile math laboratory. Besides helping to provide an ideal atmosphere for learning mathematics, the math lab will provide needed classroom space, small group study areas, and make possible a calculator laboratory that can be shared by all the schools in the program. There is a calculator for each pupil work space in the mobile lab.

The traveling teaching team spends the entire day one day each week at a particular school. Each class of twenty-four students is broken down into modules of three with one teacher or teacher aide with each module. The first few weeks strengthened the belief that "three" was the ideal number with each module. The role of the teacher aide is that of "big brother" or "big sister." He joins in the experiments as an interested partner, assists his group by asking questions, and provides limited direction to the activities being undertaken. Each teacher and teacher aide makes a special effort to develop a warm personal relationship with his charges.

The major objectives of the program are to be evaluated by pre-test and post-test scores on a mathematics achievement test, consideration of attendance records, and by questionnaires concerning attitudinal and behavioral changes. With the program underway for only a few weeks, with many equipment items still to come, and with the traveling teaching team operating in conference rooms and on auditorium stages while the mobile math lab is under construction, the following observations may give some indication of progress:

1. A teacher received a valentine from a student in the program

---

*Under-achievers are considered to be one year or more retarded in mathematics with the capacity of at least grade level achievement.

who, last semester, was her "problem child" in a regular class—indicating a dramatic change in behavior.

2. The counselor noticed that a girl who was truant returned to school in time for her math class.
3. The students in the Senate Bill 28 classes of the target classes are the envy of the other mathematics classes. Numerous statements, such as: "Can't we do what that class is doing?", "Lucky stiffs!", and "Can I get in one of those classes?", are heard frequently. A school administrator stated that the program was improving teachers' attitudes toward teaching classes of underachievers.
4. Students in the program have asked if their parents could visit class.
5. Students are beginning to ask for additional work assignments.
6. After the initial laboratory session, non-cooperative attitudes improved greatly. Students, in general, are reluctant to leave the lab at the close of the period.
7. A weekly activity (contest, puzzle, etc.) is used to introduce the laboratory lesson and to provide the opportunity to gain personal recognition. Students value greatly the ribbons bestowed on these occasions.
8. A principal stated that the eighth grade California Senate Bill 28 Mathematics Program in his school sold his ninth graders on the Senate Bill 28 Reading Program which started several weeks later.

Submitted by:
Marvin L. Johnson
Project Director
Lewis A. Prilliman
Coordinating Teacher

YOU MAY WRITE TO THE FOLLOWING FOR INFORMATION AND MATERIAL RELATED TO THE LOW ACHIEVERS IN MATHEMATICS

Superintendent of Documents U. S. Government Printing Office Washington, D.C. 20402	*The Low Achiever in Mathematics* OE 29061 Bulletin 1965, #31 Washington Conference 3/1964 Prepared by Lauren G. Woodby
National Council of Teachers of Mathematics 1201 16th St., N. W. Washington, D.C.	Request Materials Related to the Mathematics Instruction of the Low Achiever.
CAMP Central College Pella, Iowa	Concepts and Applications of Mathematics Project, Dr. Paul C. Rosenbloom, Columbia Teachers College, Director. A project for Low Achievers.
Gene Catterton State NCTM Director Wynne Public Schools Wynne, Arkansas	*Drop-in Mathematics.* 500 pages of materials compiled by 8 teachers for low achievers in 9th and 10th grades. August 1966.

Terrence G. Coburn Mathematics Coordinator Muskegon Public Schools Muskegon, Michigan	*Kit Mathematics.* A Laboratory Approach for Junior High Low Achievers. August 1966.
Joseph Lancaster Mathematics Consultant Dallas Independent School Dist. Dallas, Texas	*Structural Principles of Arithmetic* Associating understanding of principles involved to increase and apply skills in Arithmetic computations. Junior and Senior High Low Achievers.
A. Wilson Goodwin Supervisor of Mathematics Des Moines Public Schools Des Moines, Iowa	*Low-Achiever Motivational Program* (LAMP) Written and edited by Zimmerman from materials developed by a team of teachers on the Des Moines Project. Grades 8, 9, 10.
Central University of Iowa Press Pella, Iowa	*Experimental 9th Grade General Mathematics* by Shoemaker and Groenendyk (9th and 10th grades) *Advanced General Math* by Groenendyk. A course aid guide for non college bound 12th graders.
Joseph C. Glorioso Lafayette Parrish School Board P. O. Drawer 2158 Lafayette, Louisiana	Material developed by teachers for 9th grade low achievers. Utilizes a laboratory approach and incorporated beginning algebra and informal geometry.
Dr. Dave Wells Director, Instruction and Mathematics Education Oakland County Schools Pontiac, Michigan	A review of literature and research related to the teaching of General Mathematics in grades nine through twelve.

SECTION V

Mathematics for the Talented Student

Introduction

One may characterize the talented student as one who has the power to go far beyond the point at which many are forced to stop because of a lack of ability to see relations, make generalizations and acquire new, abstruse concepts. If this ability exists in the area of mathematics, it often exists in many other areas as well. To give them the best education, talented students must be identified as early as possible. This is not the task of the mathematics teacher alone; instead she is a part of a team composed of teachers of other subjects, members of the guidance staff, psychologists if available, supervisors, administrators and others. The testing program, in spite of criticisms which have been made of it, will help identify those who can profit from special attention for talented children, but only a competent teacher in collaboration with other teachers can discover the intangible and elusive elements which make for creativity in an individual.

Teachers are concerned with the identification of talented students, but, as an aid in this identification, they must also have an understanding of the basic concepts of mathematics and acquire a comprehension of the nature of creativity and talent in order to do much for such students. This involves really getting to know students, to see how they think and act, and to find ways of appealing to them to go beyond the efforts of many boys and girls. It demands something within the teacher which will not be satisfied with less than good, productive work on the part of the pupils commensurate with their capacities. Such understanding can be developed by any competent teacher who is dedicated to the improvement of instruction and most teachers are eager to improve their teaching skills.

Patterns for Teaching the Talented

There are really no set patterns for teaching talented students. The problem can be characterized by its variability. This is true because of the variability of talent, even at this level, the background of the teacher and, most unpredictable of all, the driving interests of the children. Since learning at this level, particularly, must be an individual process on the part of the pupil, the suggestions made here are for consideration whenever they can be used by teachers.

Talented students fall roughly into two categories; this is also true to some extent of other kinds of students as well. First, are the interested, well-motivated students, who have somehow acquired a basic interest in learning, probably for its own sake. That is, they are apt and hungry. The problem here is to keep them nutritiously fed, but not satiated with too rich a diet which will kill their appetites. Second, there are those who have aptitude, but no appetite, perhaps because their primary interest lies in other fields, such as, science, social studies and the like. For those in the second category the major problem is to find the right appetizer.

The classroom situations in the schools breaks down into those where there is ability grouping and those where there is not. Even so, there will be wide individual differences and teachers must find ways to cope with this situation. Approaches common in the schools involve those where the children are accelerated (an easy way to manage the program administratively) and those where the teacher is expected to emphasize depth. In either case, the teachers must adjust to the policies involved.

Suggestions for Interested Students

Many, or even most, of the aids to talented students, such as: pursuing the fundamental structure of mathematics, questioning why one takes this approach instead of that or what happens if one does something else, stimulating students to ask their own questions, providing scientific definition and terminology with clearly stated assumptions as well as reasons used in a conclusion and emphasizing the freedom and creative nature of mathematics, are all points to which students who have difficulty with mathematics are just as entitled to as is the talented student. However, the teacher should expect the talented one to do more with these ideas and to gain a greater insight into their meaning.

For the teacher who may subconsciously hold down the talented student because he is afraid of being confronted with situations with which he cannot cope, the following suggestions may be particularly pertinent.

I. Intellectual honesty is absolutely necessary. Hence, the first problem is for the teacher to ascertain, by his own self-analysis, whether or not he is intellectually honest. The fields of mathematics and logic, as well as those of the physical sciences, natural science, the social sciences and the like, to which mathematics and logic are applied, are based on intellectual honesty and this concept must be infused into our educational system at all levels.

II. Lack of the basic knowledge and ability to fully understand all of the concepts acquired by talented students is no disgrace. However, trying to cover up this deficiency by discouraging the student's interest is unworthy of a good teacher. Something should be done about it; more study, consultation with more competent people, such as, colleagues or college mathematicians and the like. Encourage students to develop individual projects and allow more freedom of the student to do his own thinking with only guidance from the teacher.

III. With all the efforts that a busy teacher can put forth, there will be times when you cannot help a particular student on a particular topic very much. Perhaps the best you can do in such a situation is to supply all the books available which might be helpful and get out of his way! There have been instances in this state where exceptionally good results have come by just such a procedure. After all many of these students are smarter than we are!

IV. For those who must remain in groups with less talented students several techniques are needed. For example, the design and construction of games and other devices for specific objectives and to be used with other students as learning tools will demand certain creative ability on the part of the talented student. This should not be allowed to degenerate into mere busy work, but should involve concepts and a new point of view for students who are creating the material.

Resources for Talented Students

While the suggestions given below are basically for the well-motivated, talented student, they can be used, if used quickly, at the first sign of interest with capable students who are unmotivated.

I. *Library Facilities Available*

It is essential that an up-to-date library be available for the use of all students, but particularly for the wide ranging curiosity and creativity of the interested talented student. Federal funds are now available to the schools to improve their library holding. All teachers who teach mathematics should have a familiarity with the material in their school library. They should go one step further and suggest that useful books be added to the library. A good procedure is to keep an active list of useful books in the hands of the librarian at all times. All librarians, on occasion, have situations arise when a book order must be submitted within a short period. These orders are often filled from lists already on file from departments and individual teachers.

One useful bibliography which covers a wide selection of books for grades 7-12 with brief comments on the content of each book is issued free by the honorary mathematics club, Mu Alpha Theta. Another booklet entitled, *The High School Mathematics Library* has been compiled by William L. Schaaf.

A most effective procedure in initiating a student into the study of mathematics beyond the regular classwork is for the teacher to be able to supply the student with a choice book which he prizes highly and with which he is thoroughly familiar. This may be from his (the teacher's) personal library or from the school library. Such a book could be: Tobias Dantzig—*Number, The Language of Science* or Lillian Leiber—*The Education of T. C. Mits.*

Many times, however, the student will express an unusual interest in material with which the teacher is not familiar. (Any mathematician is conversant with only a limited segment of mathematics.) In such cases books should be available in the school library, the public library, some nearby university library, or by the student's own purchase. One should not neglect the paperback books, available in many college and university book stores. For example, Dover Publications has reissued *The Thirteen Books of Euclid,* 3 volumes, 2nd edition, Sir Thomas L. Heath. This is an outstanding classic on Greek geometry. Dover Publications have reissued a number of the classics in mathematics at prices students can afford.

For elementary school teachers and secondary school teachers who do not know how to find certain materials or who wish to help on advising students, especially on his personal purchases, some university mathematician or mathematics department should be consulted.

II. *Magazines*

The library should have available pertinent mathematical journals for both teachers and students. A list of journals suitable for high school teachers and their students is given below. A particular high school library would probably not be able to subscribe for all of them and teachers will want some not included in the library list. The journals should vary with the students served and some selection will be necessary. They are arranged somewhat in order of usefulness for students.

1. *Mathematical Log*. Journal of the National High School and Junior College Mathematics Club, Mu Alpha Theta.
2. *Mathematics Magazine*. Mathematical Association of America.
3. *The Mathematics Student Journal*. National Council of Teachers of Mathematics.
4. *The Pentagon*. Kappa Mu Epsilon.
5. *Pi Mu Epsilon Journal*. Pi Mu Epsilon.
6. *School Science and Mathematics*, P. O. Box 108, Bluffton, Ohio.
7. *Scientific American*. American Association for the Advancement of Science.
8. *Scripta Mathematics*. 186 Street and Amsterdam Ave., N. Y., N. Y. 10033.
9. *University of Oklahoma Mathematics Newsletter*.
10. *American Mathematical Monthly*. Mathematical Association of America.
11. *Recreational Mathematics Magazine*. Joseph S. Madachy.
12. and 13. *The Mathematics Teacher and The Arithmetic Teacher*. National Council of Teachers of Mathematics.
14. *Science Books, A Quarterly Review*. American Association for the Advancement of Science.

III. *Clubs and Organizations*

Professional societies, seminars and small discussion groups provide much of the stimulus, inspiration, and motivation for professional growth among mathematicians. In the same manner, mathematical clubs, formal and informal seminars, honors classes and high school research organizations are important activities to students interested in mathematics. Any high school with an active group of mathematics students can qualify for a chapter of Mu Alpha Theta and should do so. There are also other such groups. An outstanding mathematics student who likes to ferret out things for himself should be encouraged to prepare a research paper in the fall or winter, with original ideas, for presentation to the Oklahoma Junior Academy of Science in the spring. (Contact Dr. Robert C. Fite, Director of Arts and Sciences Extension, Oklahoma State University, Stillwater, Oklahoma 74074.) These papers will be judged by a group of experts and only a limited number will be selected. This adds to the honor and prestige of presenting such a paper. Such papers can also be sent to Mu Alpha Theta for the *Mathematical Log* for possible publication. (Contact Dr. Richard V. Andree, Editor, Department of Mathematics, University of Oklahoma, Norman, Oklahoma 73069.)

Informal clubs which recognize and encourage originality can be established at any level. These may consist of only a membership list kept in the corner of the room where membership is attained by some outstanding contribution, not solicited at a regular assignment. Seminars and honors classes can also be quite beneficial if the teacher and the students are enthusiastic. They should be extra-curricular, however. It is extremely important to encourage learning for the sake of satisfying the appetite for knowledge instead for the purpose of an immediate pay off in grades and credit. This point is much more efficiently made by a teacher exhibiting an uninhibited intellectual curiosity than by talking about it.

A check should be made of available contests sponsored by nearby colleges, universities and by national organizations. While those sponsored by local colleges and universities will probably be less demanding and require less originality, more students can be accommodated and some of the questions posed are rather searching ones. A letter to the Head of the Department of Mathematics will give information about contests in the college.

It is however, difficult to predict what will appeal to students. At one such contest this writer prepared on a single page the solution of the cube root of a seven digit number as it was taught in the early 1900's. This is about as useless a procedure as one can possibly find, since it is more effectively, and probably more accurately, solved by logarithms or on a slide rule. In fact, it was necessary to search out an old arithmetic book and copy it. The students seized on it with enthusiasm and the supply was exhausted before the end of the contest. Some of them said they would want to see if their teachers could do it!

In the spring of 1966 approximately 3800 high school students from Oklahoma and Arkansas took the *Annual High School Mathematics Examination* sponsored by the Oklahoma-Arkansas Section of the Mathematical Association of America. For information on the MAA examination write Dr. Lysle C. Mason, Head, Department of Mathematics, Phillips University, Enid, Oklahoma 73701. The examination is given in the spring, but your school needs to apply in the fall if it wants to participate.

There are three problem books which have proved interesting and useful to students. These are:

The Contest Problems Book. Problems From the High School Contests of MAA (the one mentioned above) compiled with solutions by C. J. Salkind of the Polytechnic Institute of Brooklyn. It covers the period 1950-1960 and contains a classified index of problems according to subject. The problems can be solved by the use of high school geometry and intermediate algebra.

The Hungarian Problem Book I and Book II. Based on the Eotvos Competitions in Hungary, 1894-1928, compiled by J. Kursckak. The problems are probably more difficult than those of the other book, but they represent a diversity of elementary mathematical fields. Solutions are included and there is a list of problems classified according to the area involved.

Summer Science Training Programs for High Ability Secondary School Students have been and are still being sponsored by the National Science Foundation. The University of Oklahoma has sponsored a Computer Science Program for talented high school students and their teachers during the school year. It meets on alternate Saturdays throughout the year and is directed by Dr. R. V. Andree. These programs should be investigated for special students among the talented group.

How to Discover Topics of Interest

While a number of topics which may create interest with students will be suggested below, the student must at least select the topic and preferably discover it for himself. In the past students have suggested that the most fruitful way to discover these problems is to continually ask questions (which somebody, perhaps the teacher, must be able to answer) until a problem is found which fits the background, ability and interest of the particular student. If a student comes up with a question with which the teacher is not familiar, then the teacher and student should consult some university mathematician by mail or in person if possible. On the basis of his answer, it should be possible to decide whether or not the student should be encouraged to pursue the problem. One fruitful source of challenge for many students is the search for a "better" or different approach to solving problems in mathematics.

The SMSG materials in mathematics have many of these "new" approaches. They are an excellent source of problems and materials for all of the high school grades.

Suggested Topics for Creating Interest in Mathematics

The topics suggested here are not intended for distribution to students in the form of lists, rather they should be treated with care so as not to discourage them. Reveal them in small doses as the students show inclination and interest. It would probably be useful to put each topic or sub-divisions of the topics on separate cards. Some references are given, but these should be checked in the school library and others should be added as students work with particular topics. A rating of the usefulness of books and articles by students working on a topic should be added from time to time.

New topics identified by the students and teacher might well be more useful and interesting to future students than those included here. Teachers are, therefore, encouraged to collect these, file them for their own use and report them to other members of the profession, for example, through the OCTM *Newsletter,* the Mu Alpha Theta *Mathematical Log* and other publications.

I. *Suggestions for Grades 1-6*

1. *Interesting Numbers*
   Prime Numbers: 2, 3, 5, 7, 11, 13, 17, 19, . . .
   Fibonacci Numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, . . .
   Pythagorean Triples: (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25), . . .

Studying Numbers to see whether they are interesting sums or products of interesting numbers. (For example 6 is called a *perfect number* because its factors 1, 2, and 3 have 6 as a sum. In like manner 28 is the next perfect number; there are others.)

References:

Dantzig. *Number, the Language of Science*
SMSG. *Mathematics for Junior High School,* Vol. I, Part I
N. N. Vorob'ev. *Fibonacci Numbers.*

William H. Glenn and Donovan A. Johnson. *Number Patterns.*

Paul C. Rosenbloom. "Recent Information of Primes," NCTM 28th Yearbook, *Enrichment Mathematics for High School.*

2. *Number Bases*
   Counting, adding, multiplying with numbers written bases other than 10.

References:

NCTM. *Enrichment Mathematics for the Grades.*

Donovan A. Johnson and William H. Glenn. *Understanding Numeration Systems.*

SMSG. *Brief Course in Mathematics for Elementary School Teachers,* (also SMSG texts grades 1-6).

3. *Finite Sets*
   Introducing union, intersection, cartesian products, subsets and number relations between them.

References:

Donovan A. Johnson and William H. Glenn. *Sets, Sentences and Operations.*

M. Scott Norton. *Finite Mathematical Systems.*

4. *Very Elementary Logic*
   Simple observations about arguing a point.

References:

Donovan Johnson. *Logic and Reasoning in Mathematics.*

Robert L. Swain. "Logic: For Teachers, For Pupils," NCTM *Enrichment Mathematics for the Grades.*

5. *Geometry*
   Drawing interesting figures with a straight edge and compass and arriving at intuitive generalizations about the figures.

References:

Donovan A. Johnson. *Curves in Space,* 1963.

M. Scott Norton. *Geometric Constructions,* 1963.

Donovan A. Johnson and William H. Glenn. *The World of Measurement,* 1961.

SMSG. *Brief Course in Mathematics for Elementary School Teachers,* (Also SMSG texts grades 1-6).

NCTM. *Enrichment Mathematics for the Grades.*

6. *Cartesian Coordinates and Graphs*
   Tracing graphs by recurrence relations and other interesting descriptions in a cartesian coordinate system.

References:

M. Scott Norton. *Geometric Constructions.*

William H. Glenn and Donovan A. Johnson. *Adventures in Graphing.*

7. *Approaches to Arithmetic*
   Finding different ways to add, subtract, multiply and divide. Performing the basic operations with Roman numerals, with

Egyptian numerals, usefulness of the abacus. Properties of the set of whole numbers under the operations of addition and multiplication.

References:

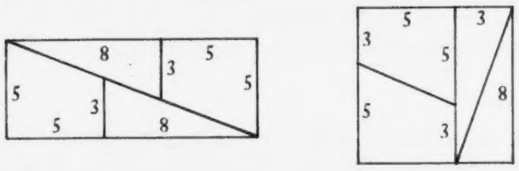
Glenn and Johnson. *Fun With Mathematics,* 1960.
Glenn and Johnson. *Short Cuts in Computing,* 1961.
Johnson and Glenn. *Computing Devices,* 1961.
Angela Pace, "Arithmetic for the Fast Learner in English Schools," NCTM 27th Yearbook, *Enrichment Mathematics for the Grades.*

II. *Suggestions for Grades 7-9.*

1. *On the Above*
Each of the above topics can be extended to the junior high school. Other, more mature, features can be included. For example, the number properties of the set of rational numbers can be investigated.

2. *Areas*
Finding areas or approximation to areas of unusual figures by cutting into squares, rectangles or triangles and adding. For example, an 8 inch square may be cut into four pieces and these refitted into a 5 × 13 inch rectangle. (See figure) This does not really prove that $64 = 65$!

Figure for Areas:



References:

Donovan Johnson and William H. Glenn. *The World of Measurement.*
NCTM, 28th Yearbook. *Enrichment Mathematics for High School.*

3. *Probability*
Many interesting intuitive probability problems can be solved at this level.

References:

Donovan Johnson. *Probability and Chance.*
J. Heller. "The Best or the Probable Best," NCTM 28th Yearbook, *Enrichment Mathematics for High School.*

4. *"Story" Problems*
Many of the classic problems are of interest to some students of the grades 7-9. This is true though the problems are useless and have *no* practical application.

References:

Irving Adler, *Magic House of Numbers,* A Signet Key Book.
Frances Hewett. "A New Look at Some Old Geometry Problems"
Charles E. Pinzka. "Problems Solving and Some Problems"
Frank S. Hawthorne. "More Non-routine Problems"
Waelaw Sierpinski. "Some Unsolved Problems of Arithmetic"
(All of the above articles are in the NCTM 28th Yearbook, *Enrichment Mathematics for High School.*

III. *Suggestions for Grades 10-12*

1. *On the Above*
   Many of the ideas above can still be extended to the high school level. For example again the number properties of the set of real numbers under the operations plus (+) and times (×) form a field and if the properties dealing with less than (<) be added, it is possible for students to prove about twenty-five basic theorems and have the background needed to develop the whole of school algebra from first year through college algebra.

References:

SMSG. *First Year Algebra,* Vol. I and II.
I. A. Barnett. *Some Ideas About Number Theory.*
Phillip Davis. *The Lore of Large Numbers.*
Herman Meyer. "Complex Numbers and Quaterions as Matrices," NCTM 28th Yearbook, *Enrichment Mathematics for High School.*

2. *Geometry*
   Construction with compass and straight edge with proofs of the constructions. Finding the errors in various methods of trisecting angles.

References:

SMSG Studies in Mathematics
  Euclidean Geometry Based on Ruler and Protractor (SM-2).
  Geometry (SM-4).
  Concepts of Informal Geometry (SM-5).
  Intuitive Geometry (SM-7).
  Applied Mathematics in High School (SM-10).
  Kazarinoff. *Geometric Inequalities.*
  Coxeter and Greitzer. *Geometry Revisited.*

3. *Convex Sets*
   Many elementary theorems are interesting and within reach of good students.

References:

Paul J. Kelly. "Plane Convex Figures" NCTM 28th Yearbook, *Enrichment Mathematics for High School.*

4. *Boolean Algebra*
   Switching circuits, called relays by electrical engineers, and Venn diagrams, even proofs from an axiomatization, are within reach and interest of some.

References:

Franz E. Hohn. *Applied Boolean Algebra, An Elementary Introduction.*
R. V. Andree. *Selections From Modern Abstract Algebra.*

5. *Probability Models*
   Ties in well with Boolean algebra and counting in finite sets.

References:

Donovan Johnson. *Probability and Chance.*
Darrall Huff and Irvin Geis. *How to Take a Chance.*

6. *Inequalities*
   Relations between harmonic, geometric, arithmetic and other means of a set of positive numbers.

References:

P. P. Korovkin. *Inequalities.*
E. Beckenback and R. Bellman. *An Introduction to Inequalities.*
N. D. Kazarinoff. *Geometric Inequalities.*

7. *Group Theory*
   Examples of permutation groups, groups of symetries and some elementary proofs.

References:

I. Grossman and W. Magnus. *Groups and Their Graphs.*
Earl, Moore and Smith. *Groups and Fields, A Programmed Unit in Modern Mathematics.*

8. *Conic Sections*
   Many interesting geometric properties can be obtained with the help of a few theorems on tangents.

References:

Donovan Johnson. *Curves in Space.*
E. H. Lockwood. *A Book of Curves.*

9. *Finite and Infinite Series*
   Many series related to arithmetic or geometric series, and many which are not, can be summed by various interesting techniques.

References:

Donald W. Hight. *A Concept of Limits.*
John F. Randolph. "Limits," NCTM 23rd Yearbook, *Insights Into Modern Mathematics.*

10. *Finite Mathematical Systems*
    Simple, limited systems in both number and geometry.

References:

John G. Kemeny, J. L. Snell and G. L. Thompson. *Finite Mathematical Structures.*
M. Scott Norton. *Finite Mathematical Systems.*

11. *Computers and Automation*
    Basic skills and theory of this fascinating, recent development in mathematics is well within the reach of good high school students. Talented students can go to much more depth without trouble.

References:

Irving Adler. *Thinking Machines.*
Edmund C. Berkeley. *Computers: Their Operation and Applications.*
Richard V. Andree. *Programming the IBM 650 Computer.*

12. *Mathematical Induction*
    Find interesting examples of the use of this type of proof from

geometry, algebra and combinatorial problems. This kind of a proof is used extensively in more advanced mathematics.

References:

I. S. Sominskii. *The Method of Mathematical Induction.*

Albert A. Blank. *Mathematical Induction,* NCTM 28th Yearbook, *Enrichment Mathematics for High School.*

13. *Topology*
    Various areas of this subject are within the reach of talented students, such as, Euler's formula for polyhedra and tracing networks (Konigsberg bridge problems, etc.).

References:

Donovan Johnson and William H. Glenn. *Topology, The Rubber-Sheet Geometry.*

Richard Courant and Herbert Robbins. "Topology" in James Newman's *The World of Mathematics,* Vol. I, pp. 581-599.

Edward Kasner and James Newman. *Mathematics and the Imagination.*

14. *Cryptanalysis*
    Secret Codes, etc. Much of this involves only high school mathematics.

References:

Lyman C. Peck. *Secret Codes, Remainder Arithmetic and Matrices.*

Suggestions for Students Without Interest

It is always a blow to the ego of a good mathematics teacher that certain really talented students do not find mathematics interesting. Not much is known about why this occurs. Perhaps it goes back to some uninspired teacher or a number of them who understood little of the nature of mathematics and succeeded only in making the subject extremely dull for their students by teaching facts, rules and tricks. In such cases the students receive little challenge, they are bored and it is not surprising that they become interested in a subject that does offer challenge and stimulation. Perhaps some minds, even the good ones, turn naturally to other areas and disciplines. We need competent people in all areas to do the work of the world.

The mathematics teachers in the schools do have one important obligation to these talented students who will go into other fields of endeavor. Mathematics is becoming increasingly important in many of these areas of study. It has always been evident that physical scientists and engineers must know mathematics, but in recent years the needs have become evident, not only in the biological sciences, but in business and management, the social sciences and other areas. Workers in all of these fields should acquire a basic understanding of mathematics before they leave the high schools and many college students, even the brilliant ones, are handicapped because they do not have it. Special extra class work by the talented student in statistics, computer programming, and other applications of mathematics to many fields might encourage these students without an appetite for the subject to acquire basic understanding which will be very helpful for their future study and work. At the present time we have no substitute for the teacher being an enthusiastic, inspiring person who knows and loves the subject and has a contagious, genuine personal interest in the overall welfare of each student.

Bibliography

I. *Books:*

1. Adler, Irving. *Magic House of Numbers.* A Signet Key Book. New York: 10022, The New American Library, 501 Madison Ave.

2. Adler, Irving. *Thinking Machines.* John Day Company, 1961.

3. Andree, Richard V. *Selections From Modern Abstract Algebra.* New York: Henry Holt Company, 1958.

4. Andree, Richard V. *Programming the IBM 650 Computer.* New York: Holt, Rinehart and Winston, 1958.

5. Barnett, I. A. *Some Ideas About Number Theory.* Washington, D.C.: 20036, NCTM, 1201 Sixteenth Street, N.W.

6. Beckenbach, E. and R. Bellman. *An Introduction to Inequalities.* Syracuse, N. Y.: L. W. Singer Co., 249-259 W. Erie Blvd., 1961.

7. Berkeley, Edmund C. *Computers: Their Operation and Applications.* Reinhold Publishing Corp., 1956.

8. Blank, Albert A. "Mathematical Induction," in *Enrichment Mathematics for High School,* 28th Yearbook of NCTM, Washington, D.C.: 20036, NCTM, 1201 Sixteenth St., N. W., 1963.

9. Coxeter, H. S. M. and S. L. Greitzer. *Geometry Revisited.* Syracuse, N. Y.: L. W. Singer Co., 249-259 W. Erie Blvd.

10. Courant, Richard and Herbert Robbins. "Topology" in James Newman, *The World of Mathematics,* Vol. 1, pp. 581-599, New York: Simon and Schuster, 1956.

11. Dantzig, Tobias. *Number, The Language of Science.* New York: MacMillan and Company, 1954.

12. Davis, Phillip. *The Lore of Large Numbers.* Syracuse, N. Y.: L. W. Singer Company, 249-259 W. Erie Blvd., 1961.

12a. Earl, Boyd, J. William Moore and Wendell I. Smith. *Groups and Fields, A Programmed Unit in Modern Mathematics.* New York: McGraw-Hill Co., Inc., 1963.

13. Glenn, William H. and Donovan A. Johnson. *Adventures in Graphing,* St. Louis, Mo.: Webster Publishing Company, 1961.

14. Glenn, William H. and Donovan A. Johnson. *Fun With Mathematics.* St. Louis, Mo.: Webster Publishing Company, 1960.

15. Glenn, William H. and Donovan A. Johnson. *Short Cuts in Computing.* St. Louis, Mo., Webster Publishing Co., 1961.

16. Glenn, William H. and Donovan A. Johnson. *Number Patterns.* St. Louis, Mo.: Webster Publishing Co., 1960.

17. Grossman, I. and W. Magnus. *Groups and Their Graphs*. Syracuse, N. Y.: L. W. Singer Co., 249-259 W. Erie Blvd.

18. Hawthorne, Frank S. "More Non-routine Problems," in *Enrichment Mathematics for High School*. 28th Yearbook of NCTM. Washington, D.C.: 20036, NCTM, 1201 Sixteenth St., N. W., 1963.

19. Heath, Thomas L. *The Thirteen Books of Euclids Elements*. New York: Dover Publications.

20. Heller, J. "The Best or the Probable Best," in *Enrichment Mathematics for High School*, 28th Yearbook of NCTM, Washington, D.C.: NCTM, 1201 Sixteenth St., N. W., 1963.

21. Hewett, Frances. "A New Look at the Same Old Geometry Problems," in *Enrichment Mathematics for High School*, 28th Yearbook of NCTM, Washington, D.C.: NCTM, 1201 Sixteenth St., N. W., 1963.

22. Hight, Donald W. *A Concept of Limits*. Englewood Cliffs N. J.: Prentice-Hall, Inc., 1966.

23. Hohn, Franz E. *Applied Boolean Algebra, An Elementary Introduction*. New York: The MacMillan Company, 1960.

24. Huff, Donald and Irvin Geis. *How to Take a Chance*. New York: W. W. Norton Company, 1959.

25. Johnson, Donovan A. and William H. Glenn. *Understanding Numerations Systems*. St. Louis, Mo.: Webster Publishing Co., 1960.

26. Johnson, Donovan A. and William H. Glenn. *Sets, Sentences and Operations*. St. Louis, Mo.: Webster Publishing Co., 1960.

27. Johnson, Donovan A. and William H. Glenn. *The World of Measurement*. St. Louis Mo.: Webster Publishing Co., 1961.

28. Johnson, Donovan A. and William H. Glenn. *Computing Devices*. St. Louis, Mo.: Webster Publishing Co., 1961.

29. Johnson, Donovan A. and William H. Glenn. *Topology, The Rubber-Sheet Geometry*. St. Louis, Mo.: Webster Publishing Co., 1960.

30. Johnson, Donovan A. *Logic and Reasoning in Mathematics*. St. Louis, Mo.: Webster Publishing Co., 1963.

31. Johnson, Donovan A. *Curves in Space*. St .Louis, Mo.: Webster Publishing Co., 1963.

32. Johnson, Donovan A. *Probability and Chance*. St. Louis, Mo.: Webster Publishing Co., 1963.

33. Kasner, Edward and James Newman. *Mathematics and the Imagination*. New York: Simon and Schuster, 1940.

34. Kazarinoff, N. D. *Geometric Inequalities*. Syracuse, N. Y.: L. W. Singer Co., 249-259 W. Erie Blvd., 1961.

35. Kelley, Paul J. "Plane Convex Figures," in *Enrichment Mathematics for High School*, 28th Yearbook of NCTM. Washington, D.C.: NCTM, 1201 Sixteenth St., N. W., 1963.

36. Kemeny, John G., J. L. Snell and G. L. Thompson. *Finite Mathematical Structures.* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1959.

37. Korovkin, P. P. *Inequalities.* New York: Blaisdell Publishing Co., A Blaisdell Scientific Paperback. 1961.

38. Kursckak, J. *Hungarian Problem Book I and Book II.* Syracuse, N. Y.: L. W. Singer Co., 249-259 W. Erie Blvd., 1961.

39. Lieber, Lillian. *The Education of T. C. Mits.* New York: W. W. Norton and Co., 1944.

40. Lockwood, E. H. *A Book of Curves.* New York: Cambridge University Press, 1961.

41. Mu Alpha Theta. *Booklist in Mathematics for Junior and Senior High School Students.* Norman, Oklahoma: Mu Alpha Theta, Box 1155, The University of Oklahoma 73069, free.

42. Norton, M. Scott. *Finite Mathematical Systems.* St. Louis Mo.: Webster Publishing Co., 1960.

43. Norton, M. Scott. *Geometric Constructions.* St. Louis, Mo.: Webster Publishing Co., 1963.

44. NCTM. *Enrichment Mathematics for the Grades.* 27th Yearbook of NCTM. Washington, D.C.: 20036. The Council, 1201 Sixteenth St., N. W., 1963.

45. NCTM. *Enrichment Mathematics for High School.* 28th Yearbook of NCTM. Washington, D.C.: 20036, The Council, 1201 Sixteenth St., N. W., 1963.

46. Pace, Angela. "Arithmetic for the Fast Learner in English Schools," in *Enrichment Mathematics for the Grades,* 27th Yearbook of NCTM. Washington, D. C.: 20036, NCTM, 1201 Sixteenth St., N. W., 1963.

47. Pinzka. "Problem Solving and Some Problems," in *Enrichment Mathematics for High School,* 28th Yearbook of NCTM. Washington, D.C.: 20036, NCTM, 1201 Sixteenth St., N. W., 1963.

48. Peck, Lyman C. *Secret Codes, Remainder Arithmetic and Matrices.* Washington, D.C.: 20036, NCTM, 1201 Sixteenth St., N. W., 1961.

49. Rosenbloom, Paul C. "Recent Information on Primes," in *Enrichment Mathematics for High School,* 28th Yearbook of NCTM, Washington, D.C.: NCTM, 1201 Sixteenth St., N. W., 1963.

50. Randolph, John F. "Limits," in *Insights Into Modern Mathematics,* 23rd Yearbook of NCTM. Washington, D.C.: 20036, NCTM, 1201 Sixteenth St., N. W., 1953.

51. Salkind, Charles T. *The Contest Problems Book,* from Annual High School Contests of MAA, with Solutions. Syracuse, N. Y.: L. W. Singer Co., 249-259 W. Erie Blvd., 1961.

52. Schaaf, William L. *The High School Mathematics Library.* Washington, D.C.: 20036, NCTM, 1201 Sixteenth St., N. W., 1963.

53. Sierpinski, Waelaw. "Some Unsolved Problems in Arithmetic," in *Enrichment Mathematics for High School,* 28th Yearbook of NCTM, Washington, D.C.: NCTM, 1201 Sixteenth St., N. W., 1963.

54. SMSG. *Mathematics for Junior High School.* Vol I, Part I. Pasadena, Calif.: 91105, A. C. Vroman, Inc., 367 South Pasadena Ave.

55. SMSG. *Brief Course in Mathematics for Elementary School Teachers.* Pasadena, Calif.: 91105, A. C. Vroman, Inc., 367 South Pasadena Ave.

56. SMSG Studies in Mathematics. *Euclidean Geometry Based on Ruler and Protractor* (SM-2); *Geometry* (SM-4); *Concepts of Informal Geometry* (SM-5); *Intuitive Geometry* (SM-7); *Applied Mathematics for High School* (SM-10). Pasadena, Calif.: 91105, A. C. Vroman, Inc., 367 South Pasadena Ave.

57. Vorob'ev, N. W. *Fibonacci Numbers.* New York: Blaisdell Publishing Co., Blaisdell Scientific Paperbacks, 1951.

II. *Magazines*

58. *Mathematical Log.* Journal of the National High School and Junior College Mathematics Club, Mu Alpha Theta (free to member institutions). Norman, Oklahoma: 73069, University of Oklahoma, Box 1155.

59. *Mathematics Magazine.* Buffalo, N. Y.: 14214, Mathematical Association of America, State University of New York.

60. *The Mathematics Student Journal.* Washington, D.C.: 20036, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W.

61. *The Pentagon.* Mount Pleasant, Michigan: Kappa Mu Epsilon, Central State College.

62. *Pi Mu Epsilon Journal.* St. Louis, Missouri: Pi Mu Epsilon, St. Louis University.

63. *School Science and Mathematics.* P. O. Box 108, Bluffton, Ohio.

64. *Scientific American.* 415 Madison Avenue, New York, 10017.

65. *Scripta Mathematica.* 186th Street and Amsterdam Avenue, New York, 10033.

66. *University of Oklahoma Mathematics Newsletter.* Department of Mathematics, University of Oklahoma, Norman, Oklahoma 73069.

67. *American Mathematical Monthly.* Mathematical Association of America, State University of New York, Buffalo, N. Y. 14214.

68. *Recreational Mathematics Magazine.* Joseph S. Madachy, P. O. Box 35, Kent, Ohio.

69. *The Mathematics Teacher and The Arithmetic Teacher*. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington, D.C., 20036.

70. *Science Books, A Quarterly Review*. American Association for Advancement of Science, 1515 Massachusetts Avenue, N. W., Washington, D.C., 20005.

SECTION VI

General Bibliography

New Mathematical Library

Monograph Project
of the
School Mathematics Study Group

1. Niven, Ivan. *Numbers: Rational and Irrational.*
2. Sawyer, W. W. *What is Calculus About?*
3. Beckenbach, E. and R. Bellman. *Introduction to Inequalities.*
4. Kazarinoff, N. D. *Geometric Inequalities.*
5. Salkind, Charles T. *The Contest Problems Book.* Problems from the Annual High School Contests of the Mathematical Association of America, with solutions.
6. Davis, P. J. *The Lore of Large Numbers.*
7. Zippin, Leo. *Uses of Infinity.*
8. Yaglom, I. M. *Geometric Transformations.* Translated from the Russian by Allen Schields.
9. Olds, C. C. *Continued Fractions.*
10. Ore, Oystein. *Graphs and Their Uses.*
11. Eötvös Competitions, 1894-1928. *Hungarian Problem Book I.*
12. __________________. *Hungarian Problem Book II.*
13. Aaboe, Asger. *Episodes From the Early History of Mathematics.*
14. Grossman, I. and W. Magnus. *Groups and Their Graphs.*
15. Niven, Ivan. *The Mathematics of Choice* (How to Count Without Counting).

These books are published by Random House and the L. W. Singer Company, 249-259 Erie Blvd., Syracuse, N. Y., 1961.

Popular Lectures in Mathematics Series

Editors: I. N. Sneddon and M. Stark

16. Sominskii, I. S. Vol. 1, *The Method of Mathematical Induction.*
17. Vorob'ev, N. N. Vol. 2, *Fibonacci Numbers.*
18. Upenskii, V. A. Vol. 3, *Some Applications of Mechanics to Mathematics.*
19. Kostovskii, A. N. Vol. 4, *Geometrical Constructions Using Compasses Only.*

20. Smogorzhevskii, A. S. Vol. 5, *The Ruler in Geometrical Constructions.*

21. Korovkin, P. P. Vol. 6, *Inequalities.*
These books, called Blaisdell Scientific Paperbacks, are published by the Blaisdell Publishing Company, a division of Random House, New York, N. Y.

Exploring Mathematics on Your Own

22. Johnson and Glenn. *Invitation to Mathematics.*

23. Johnson and Glenn. *Understanding Numeration Systems.*

24. Johnson and Glenn. *Number Patterns.*

25. Johnson and Glenn. *The World of Measurement.*

26. Johnson and Glenn. *Topology, The Rubber-Sheet Geometry.*

27. Johnson and Glenn. *The Pythagorean Theorem.*

28. Johnson and Glenn. *The World of Statistics.*

29. Johnson and Glenn. *Adventures in Graphing.*

30. Johnson and Glenn. *Computing Devices.*

31. Glenn and Johnson. *Fun With Mathematics.*

32. Johnson and Glenn. *Sets, Sentences and Operations.*

33. Glenn and Johnson. *Short Cuts in Computing.*

34. Norton, M. Scott. *Geometric Constructions.*

35. Johnson, D. A. *Curves in Space.*

36. Norton, M. Scott. *Finite Mathematical Systems.*

37. Johnson, D. A. *Logic and Reasoning in Mathematics.*

38. Johnson, D. A. *Probability and Chance.*

39. Johnson and Glenn. *Basic Concepts of Vectors.*
These books are published by the Webster Publishing Company, 1154 Reco Avenue, St. Louis, Missouri and are paperbacks.

School Mathematics Study Group

Publications

Studies in Mathematics

40. *Euclidean Geometry Based on Ruler and Protractor Axioms* (SM-2).

41. *Structure of Elementary Algebra* (SM-3).

42. *Geometry* (SM-4).

43. *Concepts of Informal Geometry* (SM-5).

44. *Number Systems* (SM-6).

45. *Intuitive Geometry* (SM-7).

46. *Concepts of Algebra* (SM-8).

47. *Brief Course in Mathematics for Elementary School Teachers* (SM-9).

48. *Applied Mathematics in High School* (SM-10).

49. *Mathematical Methods in Science* (SM-11).

50. *A Brief Course in Mathematics for Junior High School Teachers* (SM-12).

51. *In-service Course for Primary School Teachers* (SM-13).

52. *Introduction to Number Systems* (SM-14).

53. *Calculus and Science* (SM-15).

54. *Some Uses of Mathematics* (SM-16).

Supplementary and Enrichment Series

55. *Functions* (SP-1)

56. *Circular Functions* (SP-2).

57. *Teacher's Commentary for SP-1 and SP-2* (SP-3).

58. *The Complex Number System* (SP-4).

59. *Teacher's Commentary for SP-4* (SP-5).

60. *The System of Vectors* (SP-6).

61. *Teacher's Commentary for SP-6* (SP-7).

62. *Non-Metric Geometry* (SP-8).

63. *Teacher's Commentary for SP-8* (SP-9).

64. *Plane Coordinate Geometry* (SP-10).

65. *Teacher's Commentary for SP-10* (SP-11)

66. *Inequalities* (SP-12).

67. *Teacher's Commentary for SP-12* (SP-13).

68. *Numeration* (SP-14).

69. *Teacher's Commentary for SP-14* (SP-15).

70. *Algebraic Structures* (SP-16).

71. *Factors and Primes* (SP-17).

72. *Teacher's Commentary for SP-17* (SP-18).

73. *Mathematical Systems* (SP-19).

74. *Teacher's Commentary for SP-19* (SP-20).

75. *Systems of First Degree Equations in Three Variables* (SP-21).

76. *Teacher's Commentary for SP-21* (SP-22).

77. *Radioactive Decay* (SP-23).
78. *Absolute Value* (SP-24).
79. *Teacher's Commentary for SP-24* (SP-25).
80. *Mathematical Theory of the Struggle for Life* (SP-26).
81. *1 + 1 = ?* (SP-27).

Reprint Series

82. *The Structure of Algebra* (RS-1).
83. *Prime Numbers and Perfect Numbers* (RS-2).
84. *What is Contemporary Mathematics?* (RS-3).
85. *Mascheroni Constructions* (RS-4).
86. *Space, Intuition and Geometry* (RS-5).
87. *Nature and History of Pi* (RS-6).
88. *Computations of Pi* (RS-7).
89. *Mathematics and Music* (RS-8).
90. *The Golden Measure* (RS-9).

Algorithms, Computation and Mathematics

91. *Student's Text*
92. *Teacher's Commentary*
93. *Fortran, Student's Text*
94. *Fortran, Teacher's Commentary*
95. *Algol, Student's Text*
96. *Algol, Teacher's Commentary*

Probability

97. *Primary Grades, Student's Text*
98. *Primary Grades, Teacher's Commentary*
99. *Intermediate Grades, Student's Text*
100. *Intermediate Grades, Teacher's Commentary*
101. *Classroom Set of Spinners for Primary Grades*
102. *Classroom Set of Spinners for Intermediate Grades*
103. *Introduction to Probability, Basic Concepts, Student's Text, Part I*
104. *Introduction to Probability, Special Topics, Student's Text, Part II*
105. *Study Guides in Mathematics* (Volume includes Algebra, Calculus, Digital Computing and Related Mathematics, Geometry, Intermediate Mathematics, Number Theory, Probability and Statistics.)

106. *Very Short Course in Mathematics for Parents.*

107. *Philosophies and Procedures of SMSG Writing Teams.*

108. *SMSG: The Making of a Curriculum.*
All of the SMSG publications can be ordered from A. C. Vroman, Inc., 367 South Pasadena Avenue, Pasadena, California 91105.

National Council of Teachers of Mathematics
General Publications

109. *Computer Oriented Mathematics,* 1963.

110. *Continued Fractions, An Introduction,* 1964. by Charles G. Moore.

111. *The Elementary and Junior High School Mathematics Library,* 1960, by Clarence E. Hardgrove.

112. *The High School Mathematics Library,* 1963, by William L. Schaaf.

113. *Five Little Stories,* 1960, by William W. Strader.

114. *How to Study Mathematics, A Hand Book for High School Students,* by Henry Swain, 1955.

115. *Mathematical Challenges: Selected Problems From the Mathematics Student Journal,* 1965.

116. *Numbers and Numerals,* by David Eugene Smith, 1937.

117. *Number Stories of Long Ago,* by David Eugene Smith, 1919.

118. *The Number Story,* by Herta T. and Arthur H. Freitag, 1960.

119. *Paper Folding for the Mathematics Class,* 1957, by Donovan A. Johnson.

120. *Secret Codes, Remainder Arithmetic, and Matrices,* 1961, by Lyman C. Peck.

121. *Topics in Mathematics for Elementary School Teachers:* Sets; The Whole Numbers; Numeration Systems for the Whole Numbers; Algorithms for Operations With Whole Numbers; Numbers and Their Factors; The Rational Numbers; Numeration Systems for the Rational Numbers; Number Sentences. 1964.

122. *Vectors in Three Dimensional Geometry,* 1961, by Abraham M. Glicksman.

NCTM Yearbooks

123. 23rd: *Insights Into Modern Mathematics,* 1953.

124. 24th: *The Growth of Mathematical Ideas, Grades K-12,* 1959.

125. 27th: *Enrichment Mathematics for the Grades,* 1963, paper.

126. 29th: *Enrichment Mathematics for High School,* 1963, paper.

127. 29th: *Topics in Mathematics for Elementary School Teachers,* 1964.
All NCTM publications can be ordered from NCTM, 1201 Sixteenth Street, N. W., Washington, D.C., 20036.

Other Books

128. Adler, Irving. *Thinking Machines.* John Day Co., 1961, paper.

129. Adler, Irving. *Magic House of Numbers.* A Signet Key Book, New York: 10022, The New American Library, 501 Madison Avenue, 1957.

130. Andree, R. V. *Selections From Modern Abstract Algebra.* New York: Henry Holt Co., 1958.

131. Andree, R. V. *Programming the IBM 650 Computer.* New York: Holt, Rinehart and Winston, 1958.

132. Berkeley, Edmund C. *Computers: Their Operation and Applications.* Reinhold Publishing Corp., 1956.

133. Courant, Richard and Herbert Robbins. "Topology" in *The World of Mathematics.* New York: Simon and Schuster, 1956, pp. 581-599.

134. Coxeter and Greitzer. *Geometry Revisited.* Syracuse, N. Y.: L. W. Singer Co., 249-259 W. Erie Blvd., 1961.

135. Dantzig, Tobias. *Number, The Language of Science.* New York: The MacMillan Co.

136. Hight, Donald W. *A Concept of Limits.* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1966.

137. Hohn, Franz E. *Applied Boolean Algebra, An Elementary Introduction.* New York: The MacMillan Co., 1960.

138. Huff, Darrell and Irving Geis. *How to Take a Chance.* New York: W. W. Norton Co., 1959.

139. Kasner, Edward and James Newman. *Mathematics and the Imagination.* New York: Simon and Schuster, 1940.

140. Kemeny, John G., J. L. Snell, and G. L. Thompson. *Finite Mathematical Structures.* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1959.

141. Lieber, Lillian. *The Education of T. C. Mits.* New York: W. W. Norton and Co., 1944.

142. Lockwood, E. H. *A Book of Curves.* New York: Cambridge University Press, 1961.

143. Monroe, M. Evans. *The Language of Mathematics.* Ann Arbor, Michigan: University of Michigan Press, 1963.

144. Roberts, J. B. *The Real Number System in an Algebraic Setting.* San Francisco, Calif.: W. H. Freeman and Co., 660 Market Street, 1962.

145. Selby, Samuel and Leonard Sweet. *Sets, Relations and Functions.* New York: McGraw-Hill Book Co., 1963, paper.

Journals For Students And Teachers

146. *Mathematical Log.* Journal of the National High School and Junior College Mathematics Club. Norman, Oklahoma: 73069, Mu Alpha Theta (free to member institutions), Box 1155.

147. *Mathematics Magazine.* Buffalo, N. Y.: 14214, Mathematical Association of America, State University of New York.

148. *The Mathematics Student Journal.* Washington, D.C.: 20036, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W.

149. *The Pentagon.* Mount Pleasant, Michigan: Kappa Mu Epsilon, Central State College.

150. *Pi Mu Epsilon Journal.* St. Louis, Missouri: Pi Mu Epsilon, St. Louis University.

151. *School Science and Mathematics.* Bluffton, Ohio: P. O. Box 108.

152. *Scientific American.* New York: 10017, 415 Madison Avenue.

153. *Scripta Mathematica.* New York: 10033, 186th Street and Amsterdam Avenue.

154. *University of Oklahoma Mathematics Newsletter.* Norman, Oklahoma: 73069, Department of Mathematics, University of Oklahoma.

155. *American Mathematical Monthly.* Buffalo, N. Y.: 14214, Mathematical Association of America, State University of New York.

156. *Recreational Mathematics Magazine.* Kent, Ohio: Joseph S. Madachy, P. O. Box 35.

157. *The Mathematics Teacher and The Arithmetic Teacher.* Washington, D.C.: 20036, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W.

158. *Science Books, A Quarterly Review.* Washington, D.C.: 20005, American Association for Advancement of Science, 1515 Massachusetts Avenue, N. W.

Recreation and Puzzles

159. Court, Nathan A. *Mathematics in Fun and in Earnest.* New York: Mentor Books, 501 Madison Avenue, 1961.

160. Gamow, George and Marvin Stern. *Puzzle—Mathematics.* New York: Viking Press, 1958.

161. Gardner, Martin. *The Scientific American Book of Mathematical Puzzles and Diversions,* Vol. 1, New York: Simon and Schuster, 1959, Vol. 2, 1961.

162. Kemeny, John G., J. L. Snell and G. L. Thompson. *Introduction to Finite Mathematics.* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1957.

163. Northrop, Eugene P. *Riddles in Mathematics: A Book of Paradoxes.* New York: D. Van Nostrand Co., 1944.

164. Polya, Georgy. *Mathematics and Plausible Reasoning.* Vols. 1 & 2, Princeton, N. J.: Princeton University Press, 1954.

165. Schaaf, William L. *Recreational Mathematics, A Guide to Literature.* Washington, D.C.: 20036 NCTM, 1201 Sixteenth Street N. W., 1963.

166. Wylie, Clarence R. Jr. *101 Puzzles in Thought and Logic.* New York: Dover Publications, Inc., 1957.

167. Yates, Robert C. *The Trisection Problem.* Baton Rouge, La.: The Franklin Press, 1942.